1. For each set below, determine whether or not it is convex and prove your answer

(a) \{ (x, y) \mid y \geq x^4, \ 2 \geq x \geq -2 \}
(b) \{ (x, y) \mid y \geq \exp(x - 1) \}
(c) \{ (x, y) \mid y \leq x(x - 2)(x - 3) \}
(d) \{ (x, y) \mid x^2 + y^2 \leq 4, \ x \geq 0 \}

2. Find the extreme points and extreme directions of each of the convex sets from problem # 1.

3. Prove that the minimum of a linear function \( c^T x \) over \( x \in C \) for a compact (closed and bounded) convex set \( C \) must be attained at an extreme point of \( C \).

4. Prove that the maximum of a convex function \( f(x) \) over \( x \in C \) for a compact (closed and bounded) convex set \( C \) must be attained at an extreme point of \( C \).