OFDMA Cross Layer Resource Control

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Outline

OFDMA Cross Layer Resource Control

Objective Functions

- System Throughput (L1), Total Transmit Power (L1)

Constraints

- Transmit Power Constraint (L1)
- Quality of Service (User Demand, Fairness), Buffer Status (L2-3)
- Stability (L2-3)

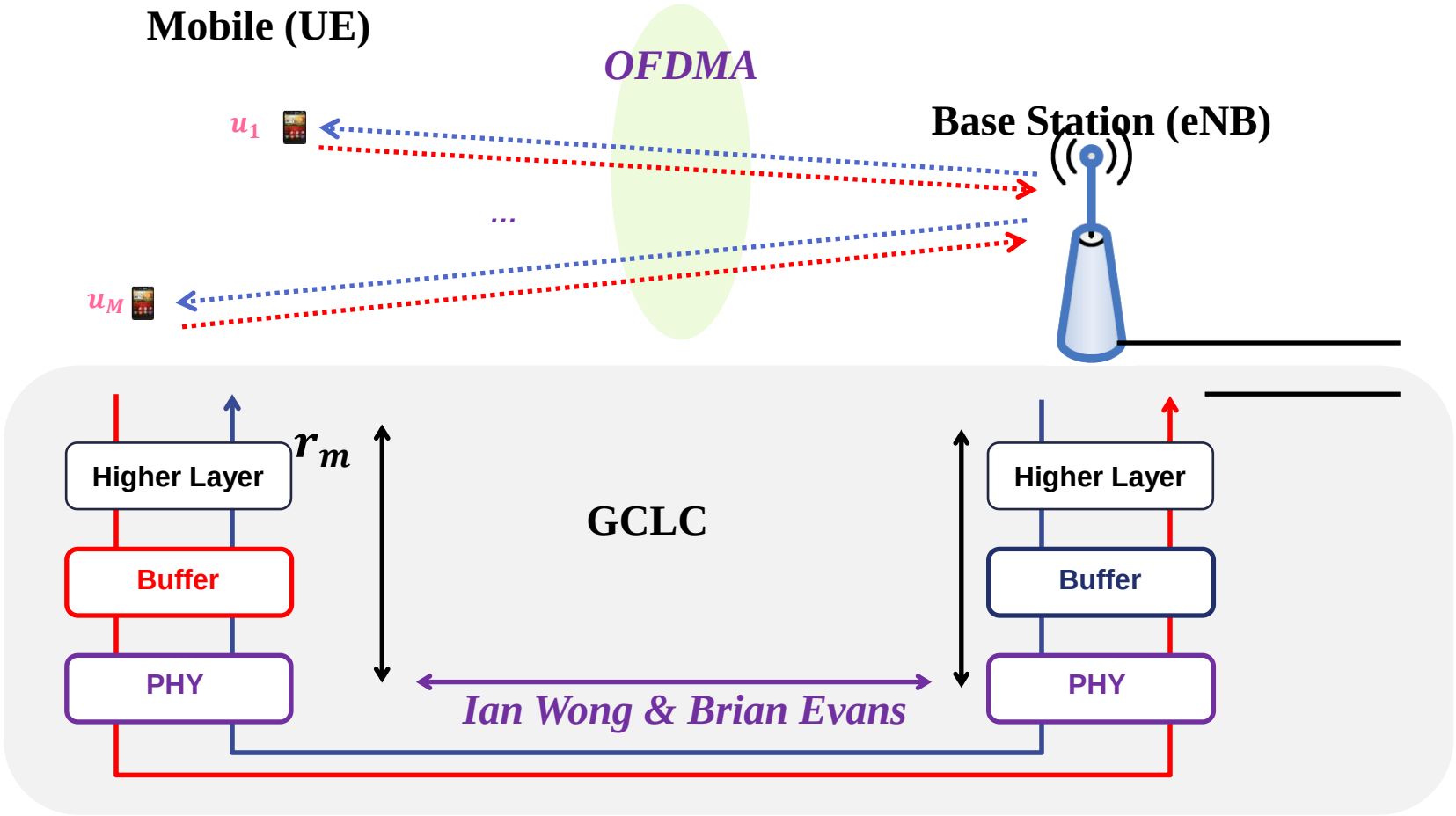
Generalized Cross Layer Control (GCLC)

Stochastic Network Optimization (SNO)

Network Utility Maximization (NUM)

OFDMA Cross Layer Resource Control

- **System Model (*M* users, *K* subcarriers)**



OFDMA Resource Control

- **Objective Functions**
  - ✓ System Throughput Maximization
  - ✓ Transmit Power Minimization
- **Constraints**
  - ✓ Transmit Power Constraint
  - ✓ Quality of Service

    User Demands : Each User Required Data Rate

    Fairness : Minimum User Data Rate
  - ✓ Stability based on Buffer Status

OFDMA Resource Allocation

- **Notations**
  - ✓ $m \in \{1, \ldots, M\}$ User Index
  - ✓ $k \in \{1, \ldots, K\}$ Subcarrier Index
  - ✓ $p_{m,k}$: Power Control Coefficient
  - ✓ $\gamma_{m,k}$: SINR for user index m and subcarrier index k
  - ✓ P_T Total Transmit Power Constraint
  - ✓ r_m Required Each User Data Rate
  - ✓ r_0 Required Minimum User Data Rate
  - ✓ b_m Buffer Service Rate
  - ✓ R_m Overall Coding Rate for User m

OFDMA Resource Allocation

- **System Throughput Maximization**
  - ✓ **Power Control**

$$p_{m,k} = \operatorname{argmax} \mathrm{E}\left\{ \overbrace{\sum_{m=1}^{M} w_m \sum_{k=1}^{K} \log(1 + p_{m,k}\gamma_{m,k})}^{\textbf{System Throughput}} \right\}$$

s.t

$$E\left[\sum_{m=1}^{M}\sum_{k=1}^{K} p_{m,k}\right] \le P_T$$

$$\max(\beta_m R_m r_0, \beta_m R_m r_m) \le w_m \sum_{k=1}^{K} \log(1 + p_{m,k}\gamma_{m,k})$$

Stability

OFDMA Resource Allocation

- **Work by Ian Wong and Brian Evans**
  - ✓ **System Throughput Maximization with Tx. Power Constraint**

$$p_{m,k} = \text{argmax}\,\mathbf{E}\left\{\sum_{m=1}^{M} w_m \sum_{k=1}^{K} \log(1 + p_{m,k}\gamma_{m,k})\right\}$$

$$\text{s.t}\quad \mathbf{E}\left[\sum_{m=1}^{M}\sum_{k=1}^{K} p_{m,k}\right] \leq P_T$$

$$\sum_{m=1}^{M} w_m = 1$$

OFDMA Resource Allocation

- **Optimization Framework**

$$L(p(\cdot),\lambda) = \mathbf{E}\left\{\sum_{m=1}^{M} w_m \sum_{k=1}^{K} \log(1 + p_{m,k}\gamma_{m,k})\right\}$$

$$+\lambda\left\{P_T - \mathbf{E}(\sum_{m=1}^{M}\sum_{k=1}^{K} p_{m,k})\right\}$$

- **Dual Optimization**

$$g^* = \min_{\lambda \geq 0} \Theta(\lambda)$$

$$\Theta(\lambda) = \max_{p(\cdot)\in P_T} L(p(\cdot),\lambda)$$

OFDMA Resource Allocation

- **Dual**

$$\Theta(\lambda) = \max_{p(\cdot)\in P_T} L(p(\cdot),\lambda)$$

$$= \lambda P_T + \max_{p(\cdot)\in P_T} \sum_{k=1}^{K} \mathbf{E}\left\{\sum_{m=1}^{M}\left[w_m \log\left(1+p_{m,k}\gamma_{m,k}\right) - \lambda p_{m,k}\right]\right\}$$

$$= \lambda P_T + \sum_{k=1}^{K} \max_{p_k(\cdot)\in P_k} \mathbf{E}\left\{\sum_{m=1}^{M}\left[w_m \log\left(1+p_{m,k}\gamma_{m,k}\right) - \lambda p_{m,k}\right]\right\}$$

$$= \lambda P_T + \sum_{k=1}^{K} E\left\{\max_{p_k(\cdot)\in P_k} \sum_{m=1}^{M}\left[w_m \log\left(1+p_{m,k}\gamma_{m,k}\right) - \lambda p_{m,k}\right]\right\}$$

$$= \lambda P_T + K E_{\gamma_k}\left\{\overbrace{\max_{m\in\{1,\ldots,M\}} \underbrace{\left[\max_{p_{m,k}\geq 0}\left(w_m \log\left(1+p_{m,k}\gamma_{m,k}\right) - \lambda p_{m,k}\right)\right]}_{\text{multilevel water filling}}}^{max\ dual\ user\ selection}\right\}$$

OFDMA Resource Allocation

- **A Simple Closed Form**

$$\tilde{p}_{m,k}(\lambda) = \left[\frac{1}{\gamma_{0,m}(\lambda)} - \frac{1}{\gamma_{m,k}}\right]^{+}$$

$$[x]^{+} = \max(0, x)$$

$$\gamma_{0,m}(\lambda) = \frac{\lambda \ln 2}{w_m}$$ Cutoff value

$$g^{*} = \min_{\lambda \geq 0}[\lambda P_T + K\, \mathbf{E}_{\gamma_k}\{g_k(\gamma_k, \lambda)\}]$$

$$g_k(\gamma_k, \lambda) = \max_{m \in \{1,\ldots,M\}}\{g_{m,k}(\gamma_{m,k}, \lambda)\}$$

$$g_{m,k}(\gamma_{m,k}, \lambda) = w_m \log\left(1 + \tilde{p}_{m,k}(\lambda)\right) - \lambda\, \tilde{p}_{m,k}(\lambda)$$

OFDMA Resource Allocation

$$g_{m,k}(\gamma_{m,k},\lambda) = w_m \log\left(1+\tilde{p}_{m,k}(\lambda)\right) - \lambda\,\tilde{p}_{m,k}(\lambda)$$

$$= \left[\frac{w_m}{\ln 2}\ln\frac{\gamma_{m,k}}{\gamma_{0,m}(\lambda)} - \frac{w_m}{\ln 2} + \frac{\lambda}{\gamma_{m,k}}\right] u(\gamma_{m,k}-\gamma_{0,m}(\lambda))$$

$$u(x) = \begin{cases} 0 & x<0 \\ 1 & x \geq 0 \end{cases}$$

- **Optimal Solution**

$$p^*_{m,k} = \tilde{p}_{m,k}(\lambda^*)1(m=m^*_k)$$

$$\tilde{p}_{m,k}(\lambda^*) = \left[\frac{1}{\gamma_{0,m}(\lambda^*)} - \frac{1}{\gamma_{m,k}}\right]^+$$

$$\lambda^* = arg\min_{\lambda\geq 0}[\lambda P + K\int_0^\infty g_k f_{g_k}(g_k)dg_k]$$

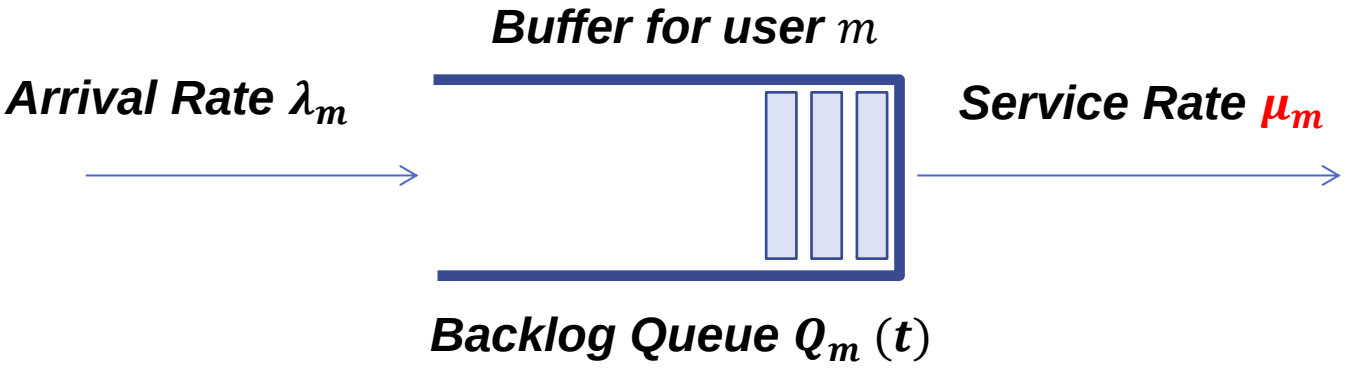
$$m^*_k = \underset{m\in\{1,\ldots,M\}}{\operatorname{argmax}}\left\{w_m \log\left(1+\tilde{p}_{m,k}(\lambda^*)\gamma_{m,k}\right) - \lambda^*\tilde{p}_{m,k}(\lambda^*)\right\}$$

Cross Layer Control

- **Generalized Cross Layer Control (GCLC)**
  - ✓ **Proposed by** Georgiadis, Neely, and Tassiulas
  - ✓ Focus on Stability based on ***Queuing Statistics***
    - **Stochastic Network Optimization**
    - **Network Utility Maximization**
  - ✓ Network Stability
    - Differential Equation of Queuing Statistics
    - **Lyapunov Stability**

Cross Layer Control

- **Stochastic Network Optimization**



Network State Variable $S(t)$

Control Action $I(t) \in I_{S(t)}$ *feasible control region under* $S(t)$

$$\mathbf{Q}(t) \xrightarrow{S(t),I(t)} \mathbf{Q}(t+1)$$

Cross Layer Control

- **Stochastic Network Optimization**

$$Q_m(t+1) \leq \max\left[Q_m(t) - R_m^{out}\left(I(t), S(t)\right), 0\right] + R_m^{in}(I(t), S(t))$$

Outgoing Queue ***Entering Queue***

- **Stability Issue**

$$\lim_{t \to \infty} \sup \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbf{E}\{Q_m(\tau)\} < \infty$$

Lyapunov Stability

If there exist $B > 0$ and $\epsilon > 0$, such that for all times slot t we have :

$$\mathbf{E}[\Delta(t)|\mathrm{Q}(t)] \leq B - \epsilon \sum_{m=1}^{M} Q_m(t)$$

Then network is strongly stable, and

$$\lim_{t \to \infty} \sup \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbf{E}\{\mathrm{Q}(\tau)\} < \frac{B}{\epsilon}$$

Cross Layer Control

- **Find $I(t)$**

$$I^*(t) = \underset{I(t)\in \mathbf{I}_{S(t)}}{\operatorname{argmax}} \sum_{ab} W_{ab}^*(t)\mu_{ab}^*(t)$$

$$W_{ab}^*(t) = \max_{\mu_{ab}} [Q_a(t) - Q_b(t)]^+$$

maximum queue backlog differential

- **Find Λ by Lyapunov Drift**

  - ✓ **Drift Definition**

$$\Delta(t) = L(t+1) - L(t)$$

$$L(t) = \frac{1}{2}\sum_{m=1}^{M} Q_m^2(t)$$

Lyapunov Drift

$$\Delta(t) = L(t+1) - L(t)$$

$$= \frac{1}{2}\sum_{m=1}^{M}\left[Q_m^2(t+1) - Q_m^2(t)\right]$$

After applying $Q_m^2(t+1)$

Find Lyapunov Bound with Conditional Expectation

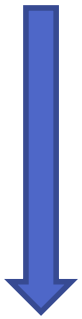
$$\{\Delta(t)|\mathbf{Q}(t)\} \leq \ \ldots$$

$$\Rightarrow \lim_{t\to\infty}\sup\frac{1}{t}\sum_{\tau=0}^{t-1}\mathbf{E}\{Q_m(\tau)\} < \infty$$

Network Utility Maximization (NUM)

- **Rate $\mathbf{r} \in \Lambda$ with Maximum Utility**

$$\mathbf{r}^* = \underset{\mathbf{r} \leq \boldsymbol{\lambda}}{\operatorname{argmax}} \{ g(\mathbf{r}) | \mathbf{r} \in \Lambda \}$$



$g(\mathbf{r})$: Utility Function

Minimize Cost

Generalized Cross Layer Control

- **General Form of GCLC**

$$\min_{\mathbf{r}\leq\boldsymbol{\lambda}}\{f(\mathbf{x})-g(\mathbf{y})|\mathbf{q}(\mathbf{x})\leq\mathbf{Q},\mathbf{h}(\mathbf{y})\geq\mathbf{H},\mathbf{r}\in\boldsymbol{\Lambda}\}$$

✓ **Cost Variable Vector $\mathbf{x}$:** Maximum cost constraints $\mathbf{Q}$

✓ **Utility Variable Vector $\mathbf{y}$:**Minimum utility constraints $\mathbf{H}$

✓ **Stable Region $\mathbf{r}\in\boldsymbol{\Lambda}$**

- **Arrival Rate Vector $\boldsymbol{\lambda}$**

✓ **Minimize net cost**

- Natural cost function $f(x)$ and Concave Utility function $g(y)$

OFDMA Resource Control via GCLC

- **OFDMA via GCLC**

$$\min_{\mathbf{r}\leq\boldsymbol{\lambda}}\{f(\mathbf{x})-g(\mathbf{y})|\mathbf{q}(\mathbf{x})\leq\mathbf{Q},\mathbf{h}(\mathbf{y})\geq\mathbf{H},\mathbf{r}\in\boldsymbol{\Lambda}\}$$

✓ **Cost Variable Vector $\mathbf{x}$: power coefficients**

- $\mathbf{Q}=\mathbf{P_T}$ **: Total Power Constraint**

✓ **Utility Variable Vector $\mathbf{y}$:** user data rate $\mathbf{y}=\mathbf{r}$

- $\mathbf{H}$: Quality of Service (User demands, Fairness)

✓ **Stable Region based on Queuing Statistics**

✓ **Minimize net cost : Maximize System Throughput**