Efficient Control Signaling for Resource Allocation

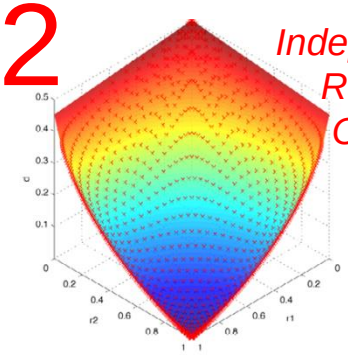
Ph.D. Thesis Defense

Gwanmo Ku

May 21, 2014

Overview

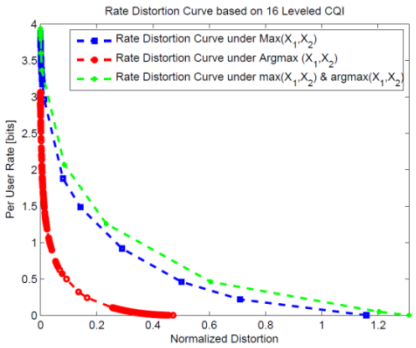
2



Independent CEO Rate Region Calculation

3

Independent CEO with Resource Allocation

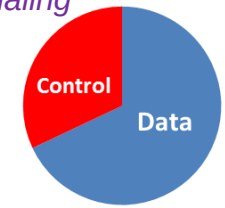


System Performance (Spectral Efficiency) for Data

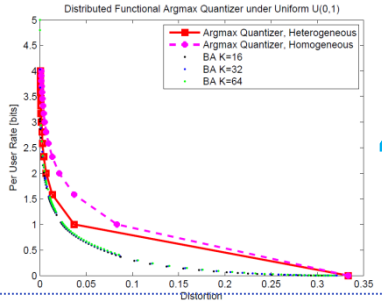
Information Theory Bound

1

LTE Control Signaling



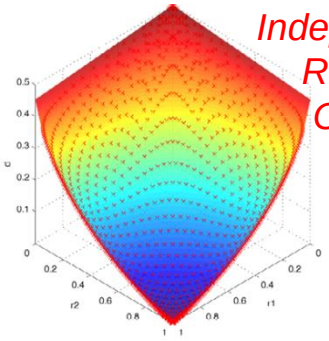
Practical Source Coding



4

Control (Collaboration) Info.

Overview

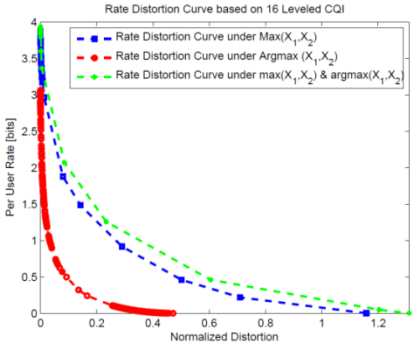


Independent CEO Rate Region Calculation

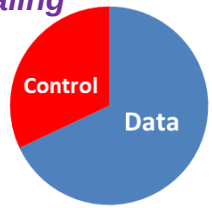
System Performance (Spectral Efficiency) for Data

Information Theory Bound

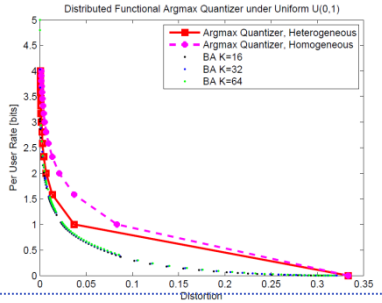
Independent CEO with Resource Allocation



LTE Control Signaling*



Practical Source Coding

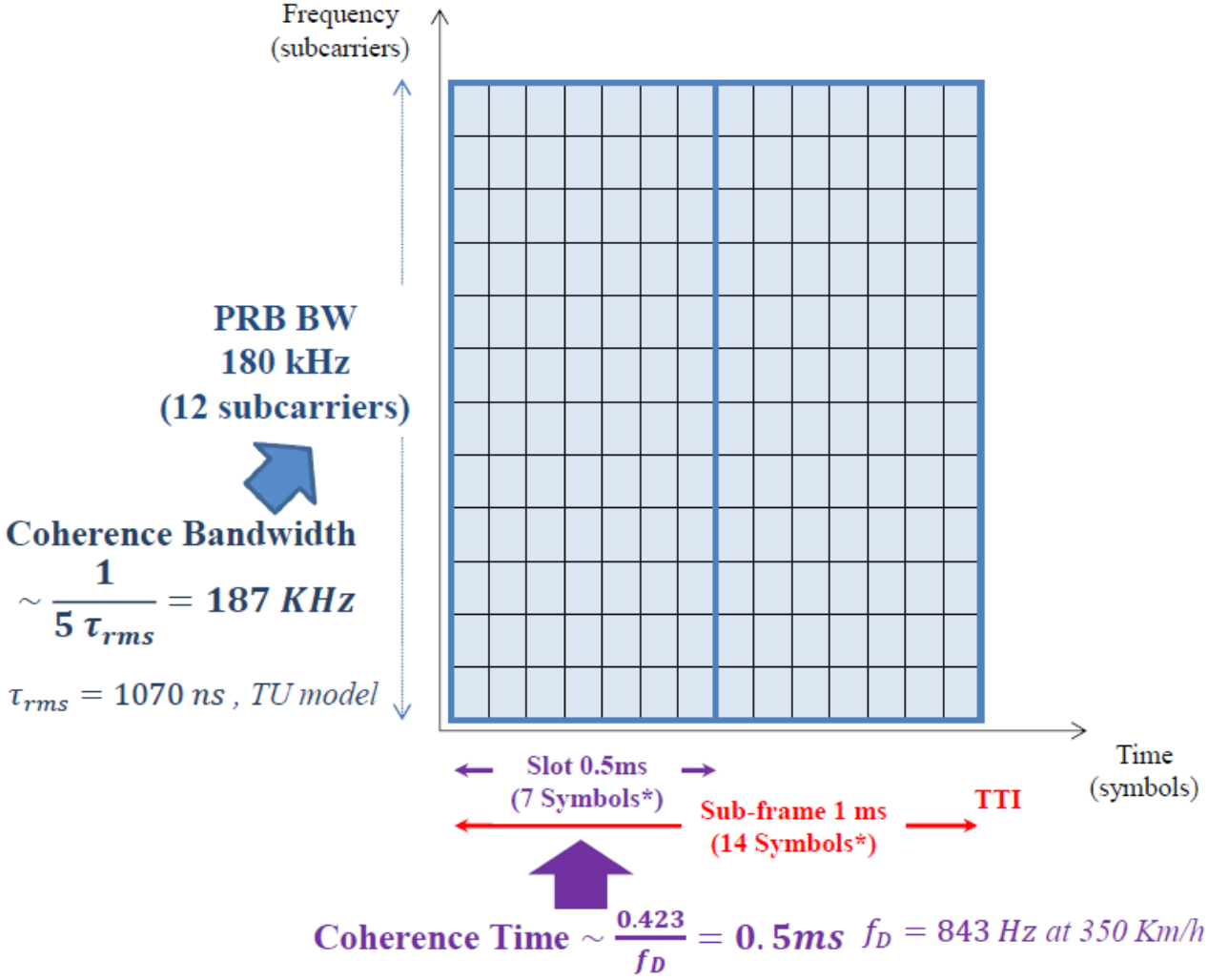


Control (Collaboration) Info.

* G. Ku, J.M. Walsh, "Resource Allocation and Link Adaptation in LTE and LTE Advanced : A Tutorial", IEEE Communications Surveys and Tutorial

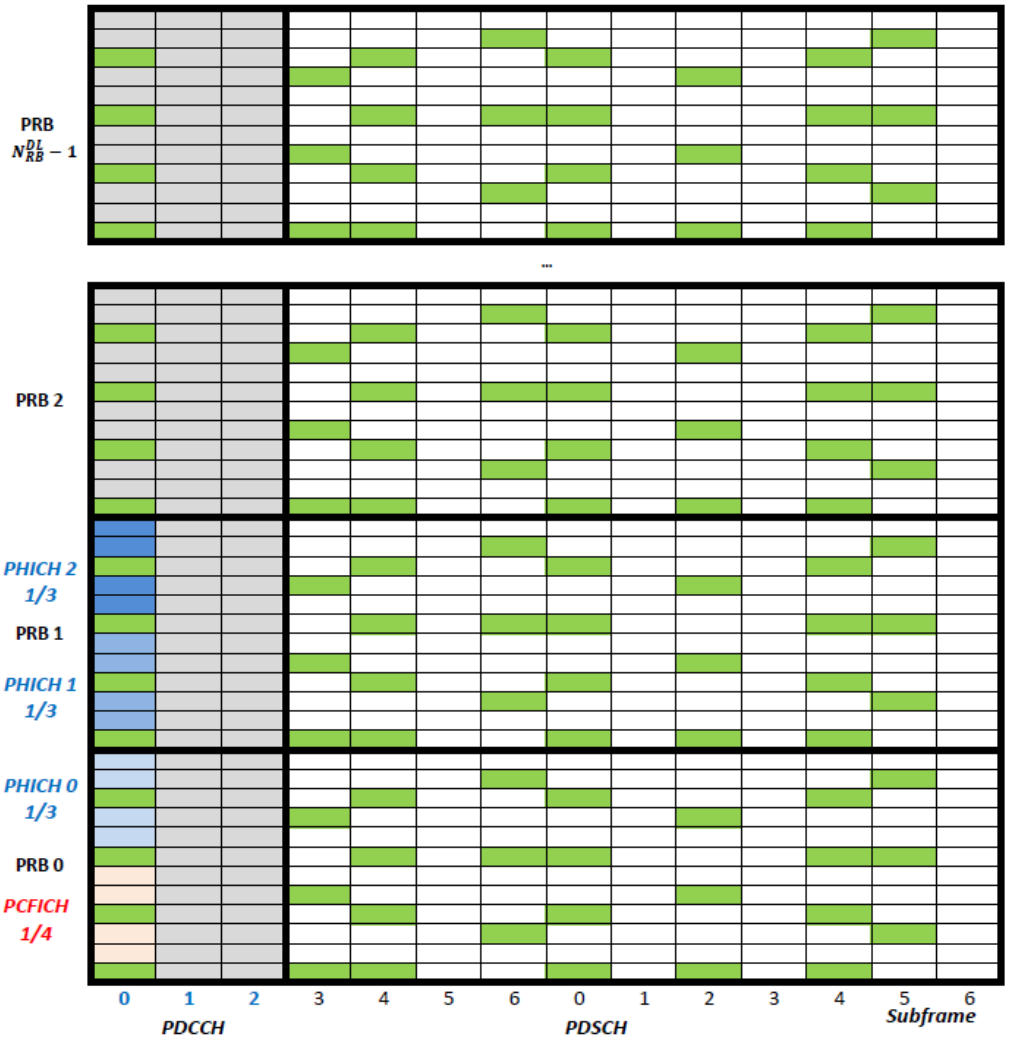
Resource Grid

Units : Resource Allocation & Scheduling



Control Region : PDCCH Region

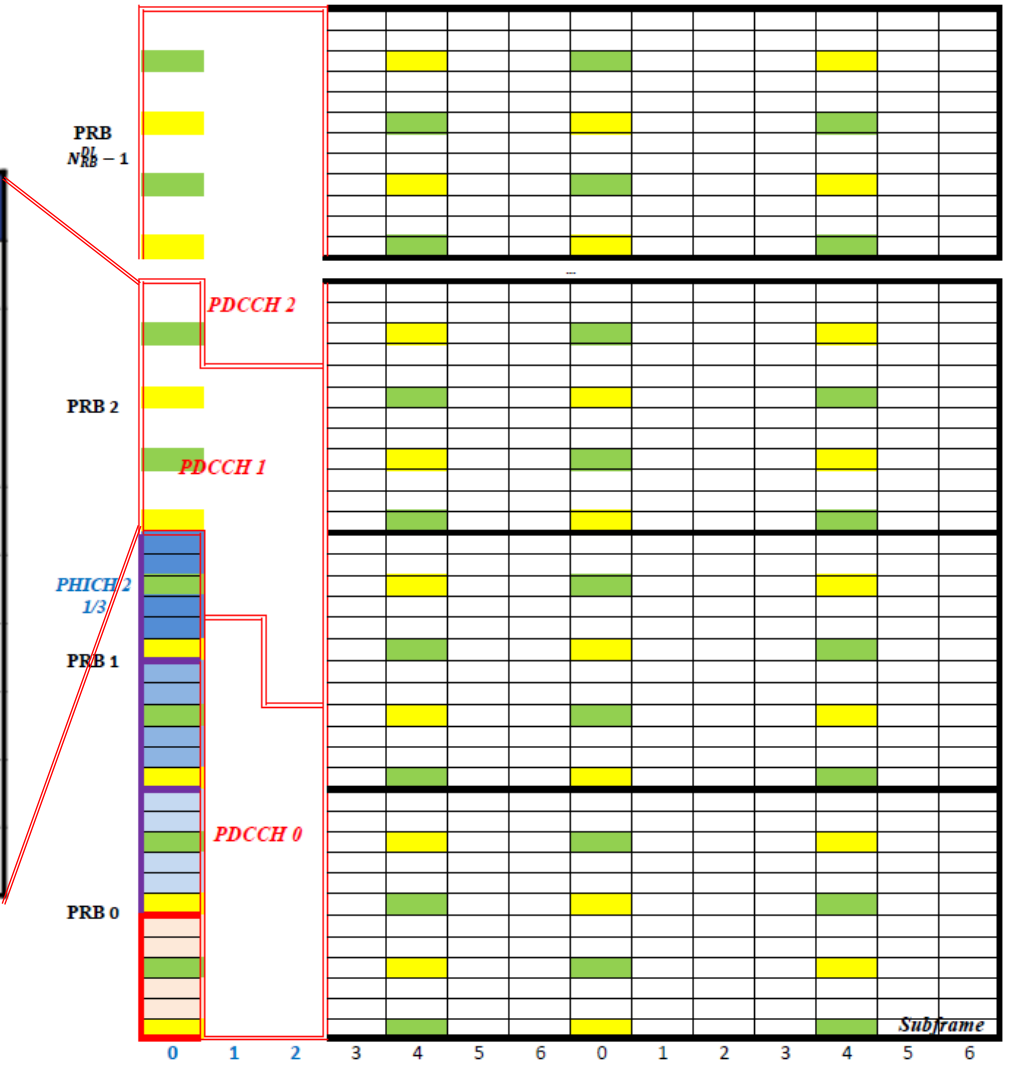
Downlink Control Region by PCFICH



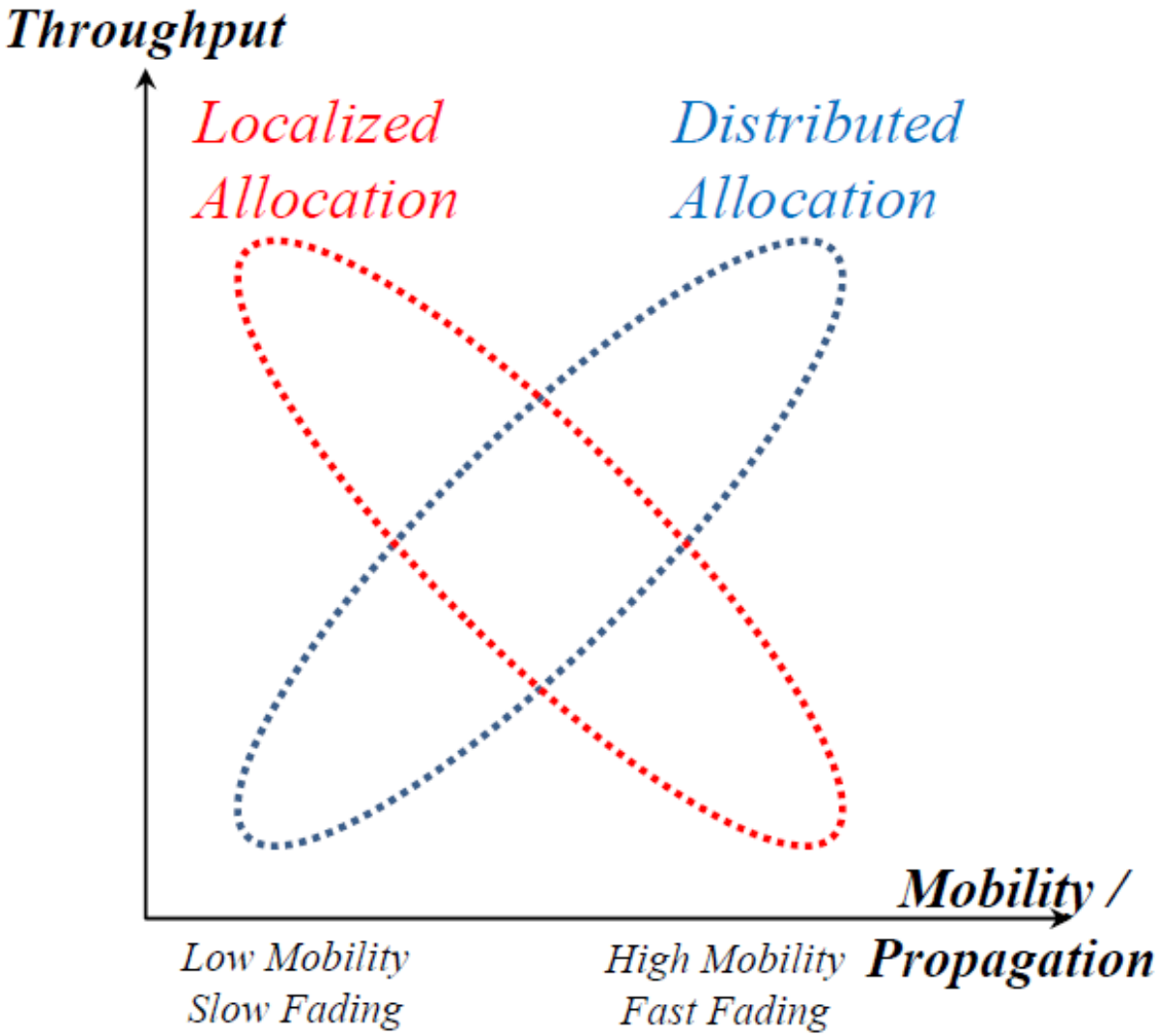
Downlink Control Information in PDCCH

Contents of Downlink Control Information

Bits	DCI Format 1
1	Resource Allocation Header: Set to 1
$\left\lceil \frac{N_{RB}^{DL}}{P} \right\rceil$	**Resource Allocation Type 1** $\lceil \log_2(P) \rceil$ **bits** : *Selected Resource Block Subset* **1 bits** : a *shift of resource allocation span* $\left\lceil \frac{N_{RB}^{DL}}{P} \right\rceil - \lceil \log_2(P) \rceil - 1$ **bits** : *Resource Assignment*
5	***MCS***
3 or 4	HARQ Process : 3 for FDD, 4 for TDD
1	New Data Indicator
2	Redundancy Version
2	UL Power Control (PUCCH)
2	Downlink Assignment Index : TDD



Resource Allocation

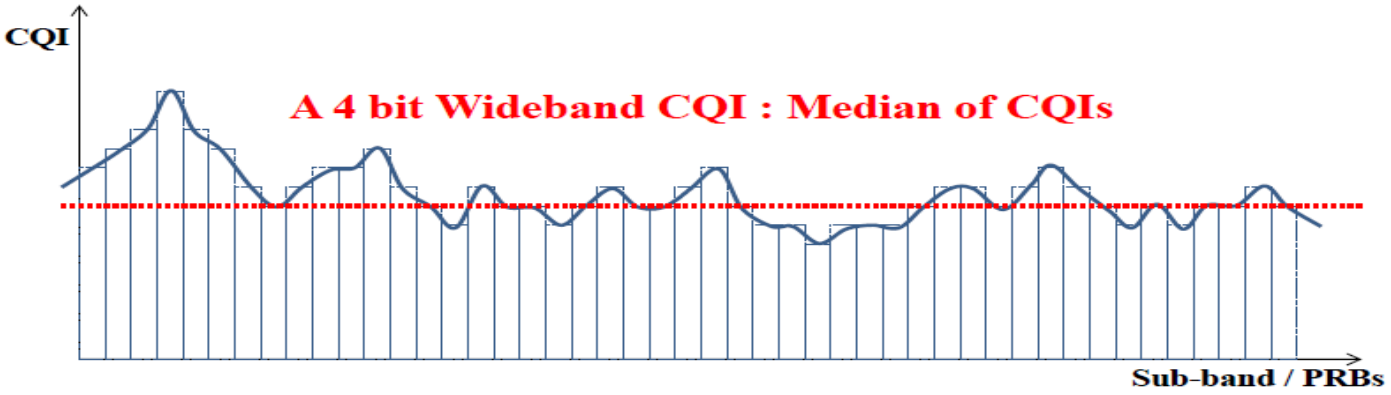


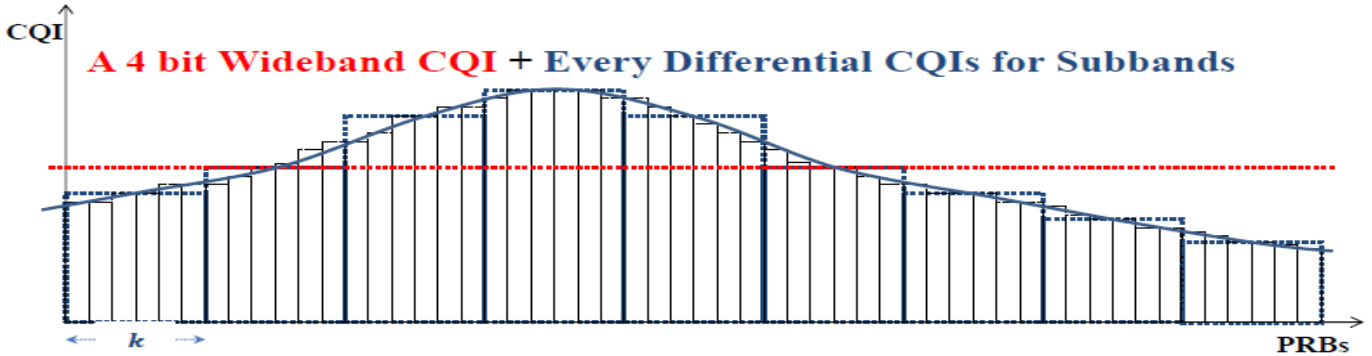
MCS Index and CQI

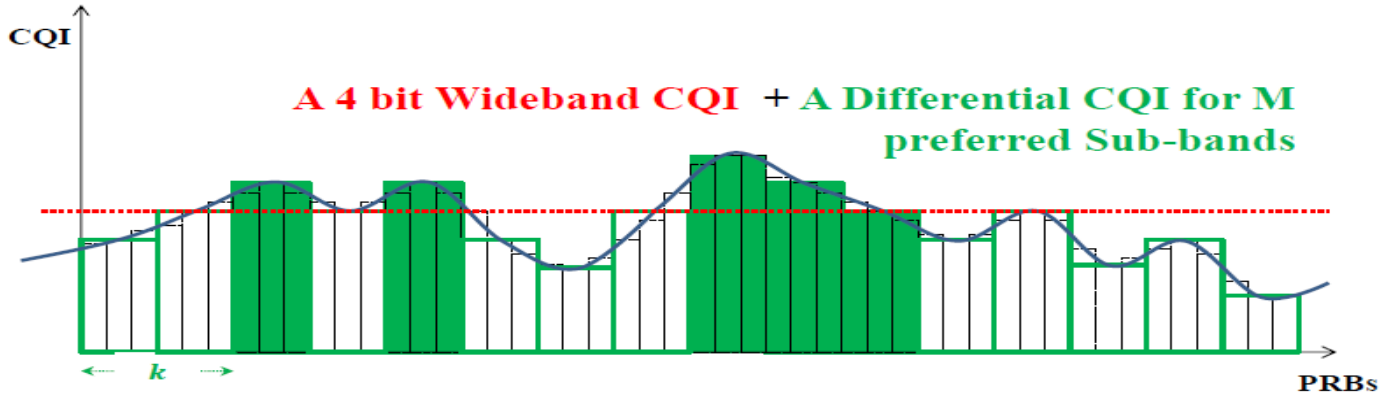
MCS	CQI	Modulation	Code rate	Spectral Efficiency
-	0	Out of range		
-	1	QPSK	0.0761	0.1523
0	2		0.1171	0.2344
1			0.1533	0.3057
2	3		0.1884	0.3770
3			0.2451	0.4893
4	4		0.3007	0.6016
5			0.3701	0.7393
6	5		0.4384	0.8770
7			0.5136	1.0264
8	6		0.5878	1.1758
9			0.6630	1.3262
10		16 QAM	0.3320	1.3262
11	7		0.3691	1.4766
12			0.4238	1.6953
13	8		0.4785	1.9141
14			0.5400	2.1602
15	9	16QAM	0.6015	2.4063
16			0.6425	2.5684
17		64 QAM	0.4277	2.5684
18	10		0.4550	2.7305
19			0.5048	3.0264
20	11		0.5537	3.3223
21			0.6015	3.6123
22	12		0.6503	3.9023
23			0.7021	4.2128
24	13		0.7539	4.5234
25			0.8027	4.8193
26	14		0.8525	5.1152
27			0.8886	5.3349
28	15		0.9257	5.5547
29		QPSK	Reserved	Reserved
30		16QAM		
31		64QAM		

Reference : 3GPP Technical Specification TS 36.213

CQI Reporting Modes



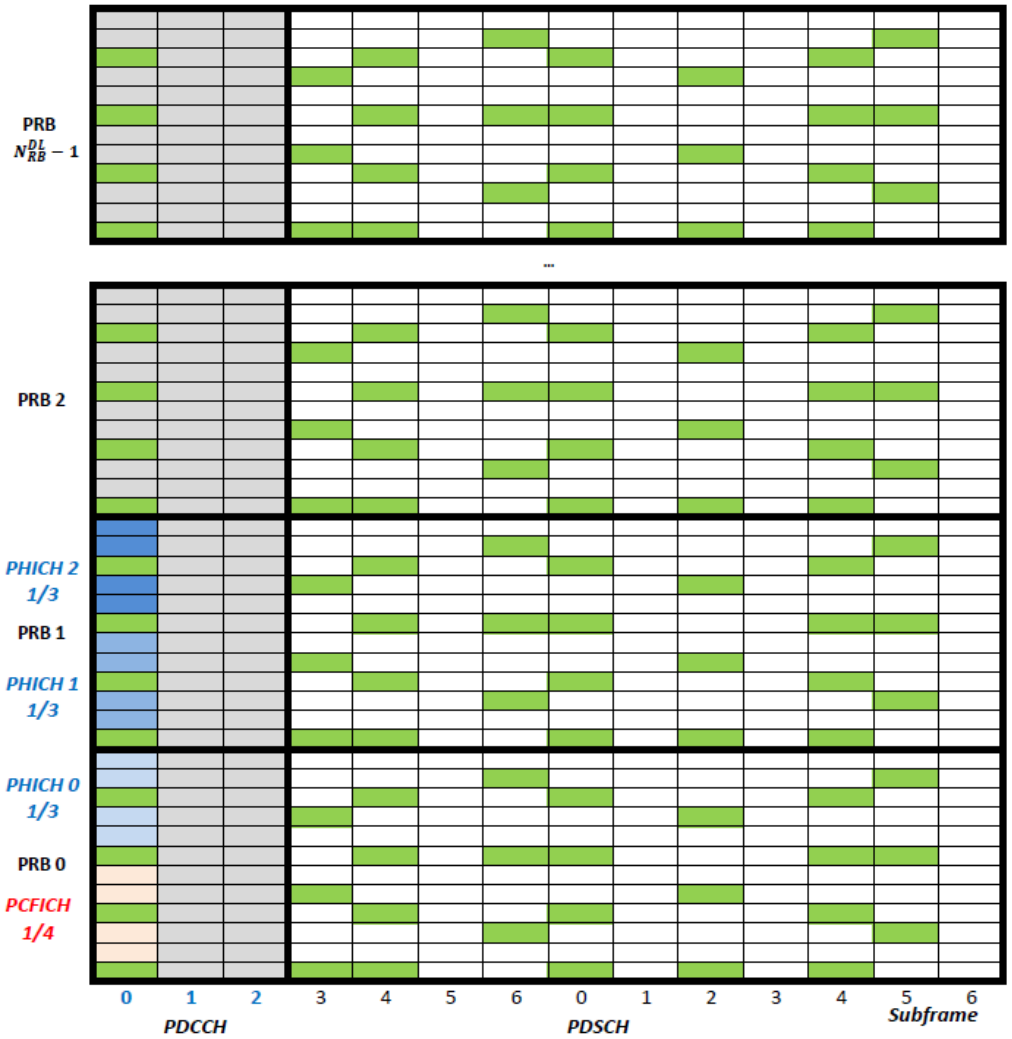




Reference : 3GPP Technical Specification TS 36.213

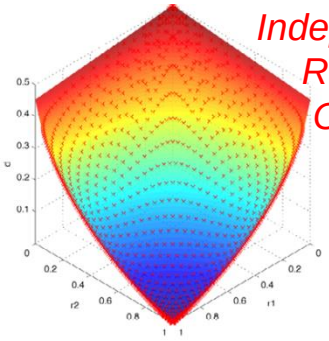
Control Signaling Overhead

Control Channels 21% **32%** **11% Reference Signals**



Reference : 3GPP Technical Specification TS 36.211

Overview



Independent CEO Rate Region Calculation

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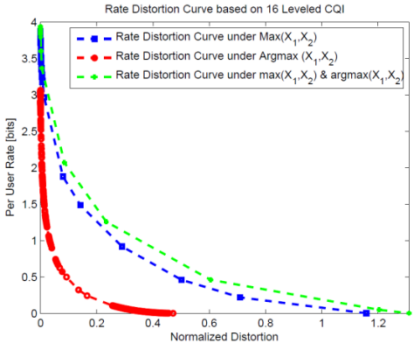
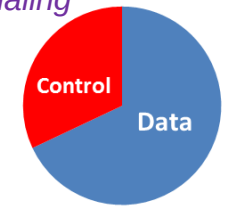
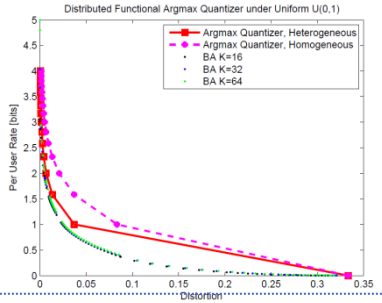


Information Theory Bound

LTE Control Signaling

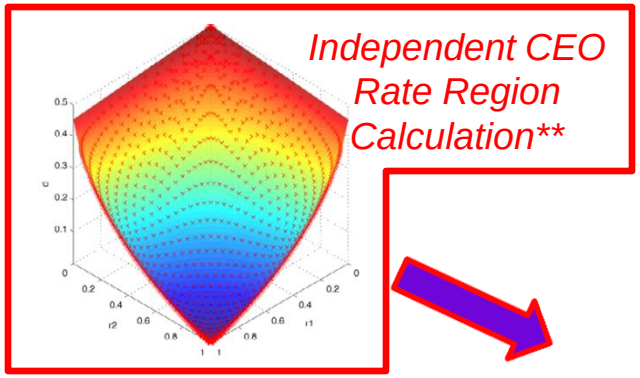


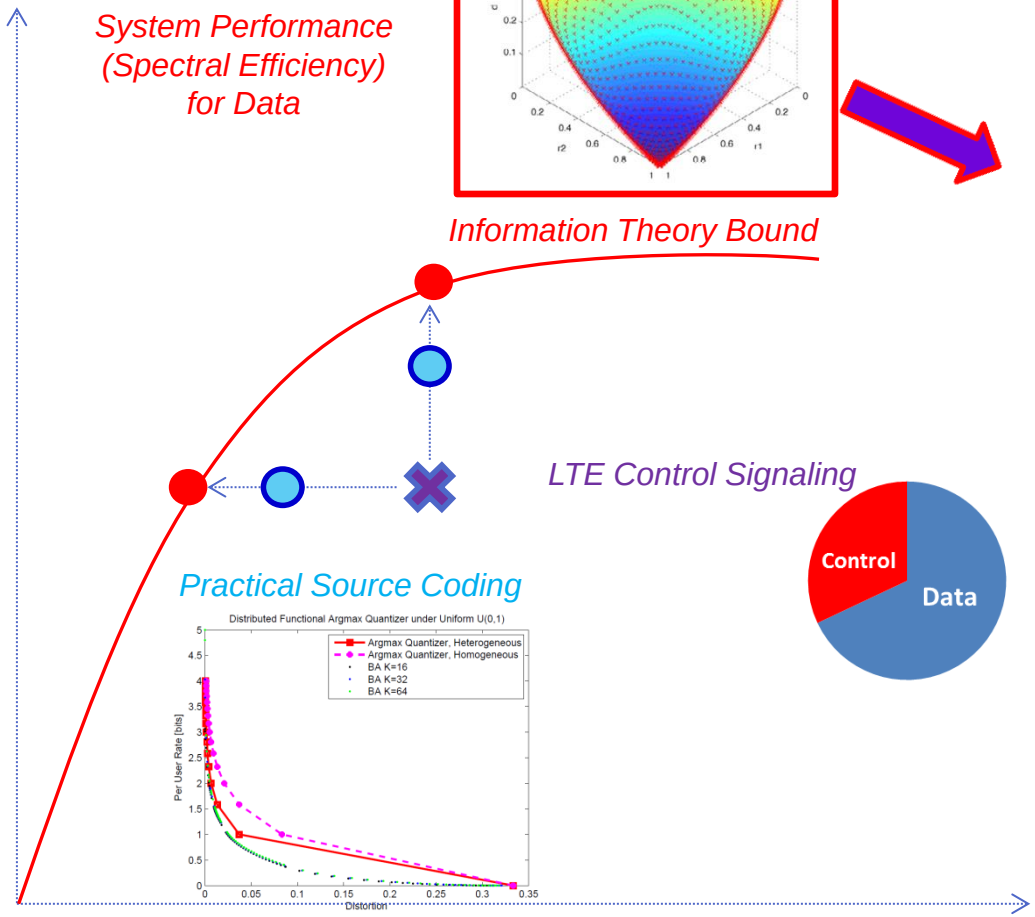
Practical Source Coding

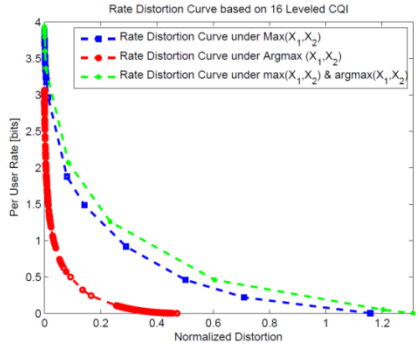


Control (Collaboration) Info.

Overview



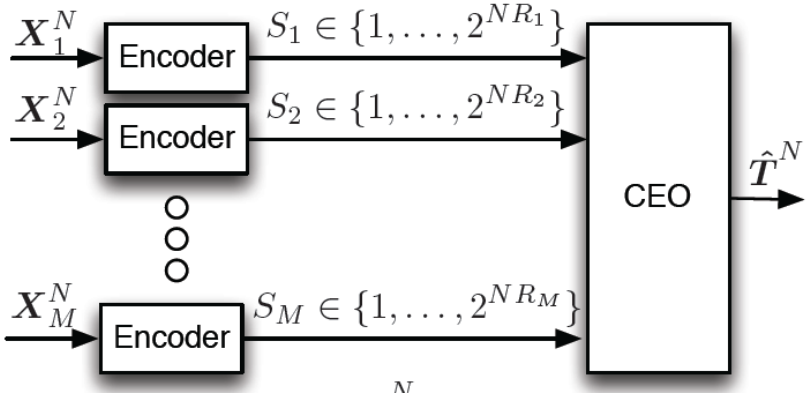




**G. Ku, J. Ren, J.M. Walsh, "Computing the Rate Distortion Region for the CEO Problem with Independent Sources", IEEE Transactions on Signal Processing

Independent CEO Model

System Diagram & Rate Region Expression



$$\left\{ (\boldsymbol{\mathcal{R}}, D) \;\middle|\; \begin{array}{c} \forall m \in \boldsymbol{\mathcal{M}} \;\; R_m \geq I(X_m; U_m) \\ U_m \leftrightarrow X_m \leftrightarrow \boldsymbol{X}_{\boldsymbol{\mathcal{M}}\backslash m}, \boldsymbol{U}_{\boldsymbol{\mathcal{M}}\backslash m}, T \\ \hat{T} \leftrightarrow \boldsymbol{U}_{\boldsymbol{\mathcal{M}}} \leftrightarrow T, \boldsymbol{X}_{\boldsymbol{\mathcal{M}}} \\ \mathbb{E}\left[d(T, \hat{T})\right] \leq D \end{array} \right\}$$

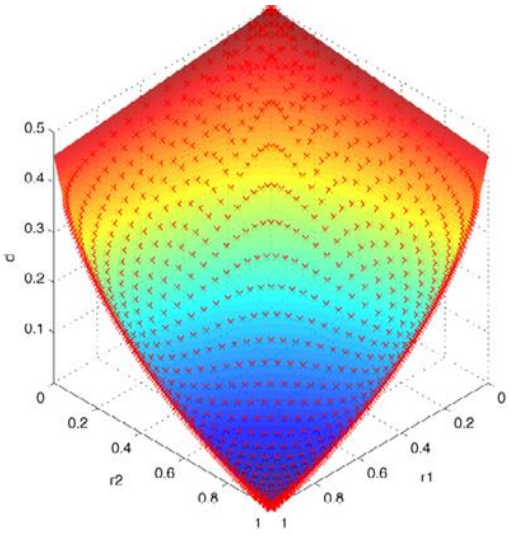
Rate Region Calculation

Calculating Rate Region

$$\left\{ (\boldsymbol{\mathcal{R}}, D) \left| \begin{array}{c} \forall m \in \boldsymbol{\mathcal{M}} \;\; R_m \geq I(X_m; U_m) \\ U_m \leftrightarrow X_m \leftrightarrow \boldsymbol{X}_{\boldsymbol{\mathcal{M}}\backslash m}, \boldsymbol{U}_{\boldsymbol{\mathcal{M}}\backslash m}, T \\ \hat{T} \leftrightarrow \boldsymbol{U}_{\boldsymbol{\mathcal{M}}} \leftrightarrow T, \boldsymbol{X}_{\boldsymbol{\mathcal{M}}} \\ \mathbb{E}\left[d(T, \hat{T})\right] \leq D \end{array} \right. \right\}$$

Algorithm

Fixed
$p_{X_m}(x_m), d$

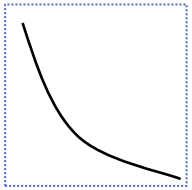


Flowchart

Calculating Rate Region

Grid of $\vec{\lambda}$ and μ

different slope →



$$\min \sum_{i \in \mathcal{M}} \lambda_i^{(k)} I(X_i; U_i) + \mu^{(k)} \mathbb{E}\left[d(T, \hat{T})\right]$$

Redundancy Removal

$$(r_i^{(k)}, d^{(k)})$$

Polyhedral Rep. conversion

$$\left\{(\vec{r}, d) | \mathbf{H}\vec{r} + \vec{h}d \geq \mathbf{0}\right\}$$

Standard BA Relaxation

Calculating Rate Region

$$\mathcal{L}_{\lambda\mu}(\boldsymbol{Q},\boldsymbol{q},\Lambda)=\sum_{i\in\mathcal{M}}\lambda_i\sum_{(x_i,u_i)}p_{X_i}(x_i)Q_i(u_i|x_i)\log\frac{Q_i(u_i|x_i)}{q_i(u_i)}$$

$$+\mu\sum_{u,x,t,\hat{t}}d(t,\hat{t})p_{T|\boldsymbol{X}}(t|\boldsymbol{x})\Lambda(\hat{t}|\boldsymbol{u})\prod_{i\in\mathcal{M}}Q_i(u_i|x_i)$$

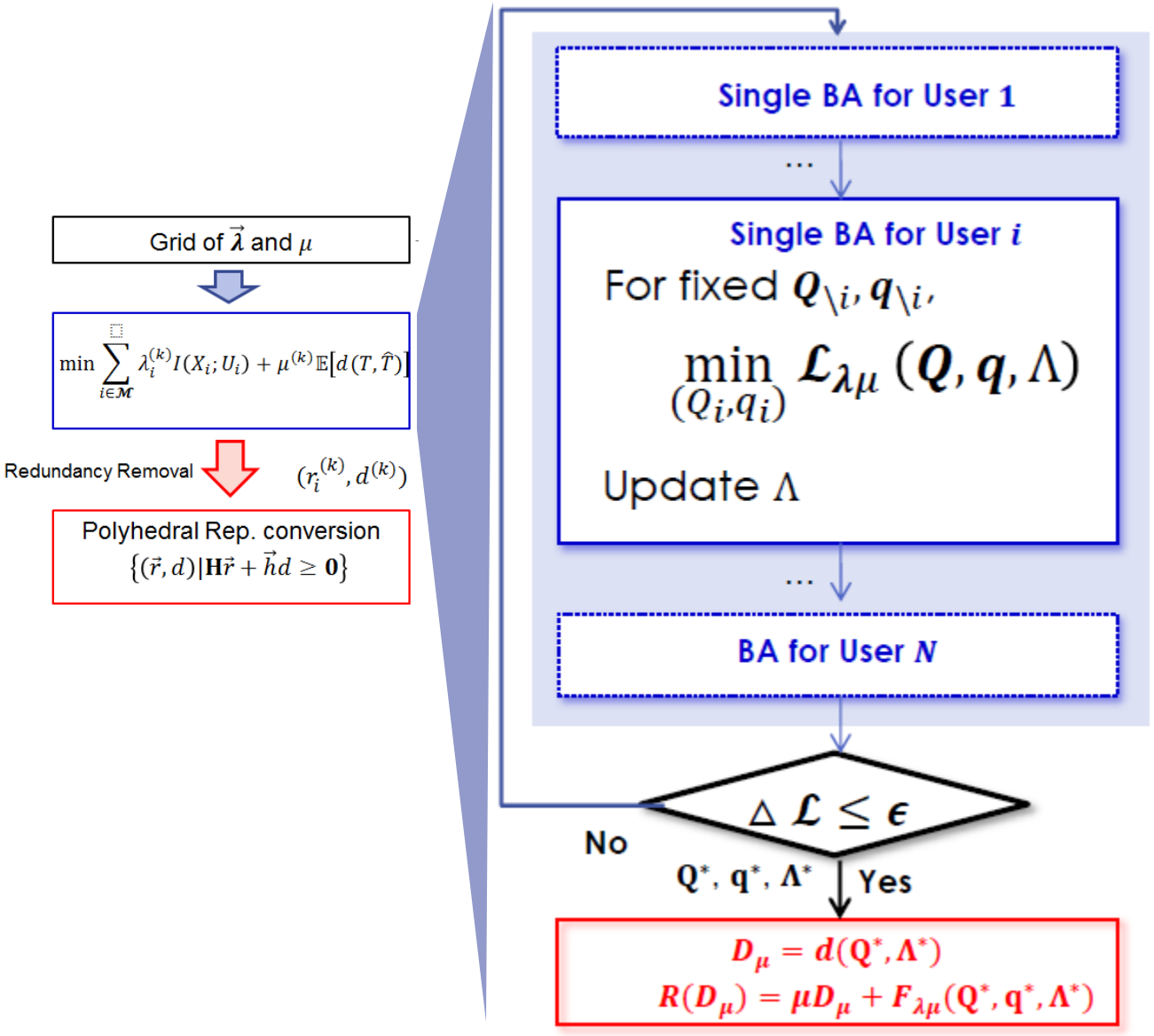
where

$$\boldsymbol{Q}\doteq[Q_i(u_i|x_i)\mid u_i\in\mathcal{U}_i,x_i\in\mathcal{X}_i,i\in\mathcal{M}]$$

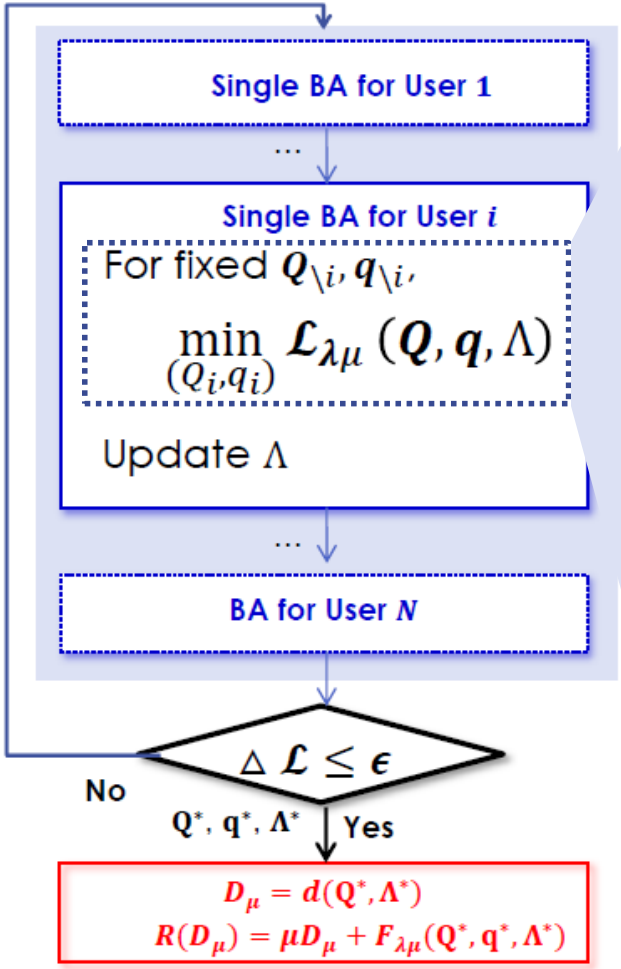
$$\boldsymbol{q}\doteq[q_i(u_i)\mid u_i\in\mathcal{U}_i,i\in\mathcal{M}]$$

$$\Lambda\doteq\left[\Lambda(\hat{t}|u)|\hat{t}\in\hat{\mathcal{T}},\boldsymbol{u}\in\mathcal{U}_i\times\cdots\times\mathcal{U}_M\right]$$

Flowchart of Calculating Rate Region



Flowchart

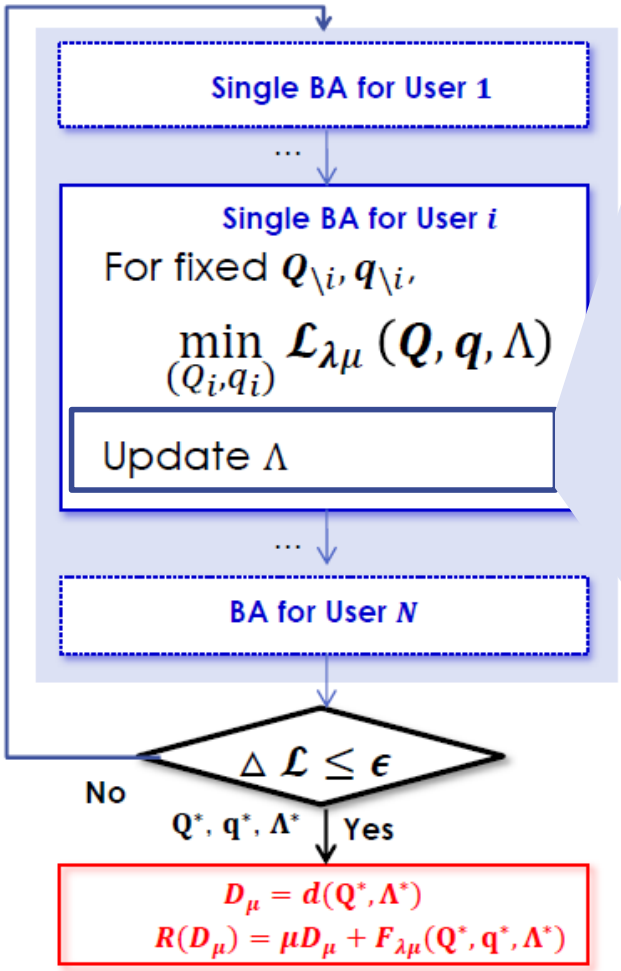


$$q_i^{(l,k_i)}(u_i) = \sum_{x_i} p_{X_i}(x_i) Q_i^{(l,k_i)}(u_i|x_i)$$

$$Q_i^{(l,k_i+1)}(u_i|x_i) = \frac{q_i^{(l,k_i)}(u_i) 2^{-\frac{\mu}{\lambda_i} d_i^{(l,i)}(x_i,u_i)}}{\sum_{u_i'} q_i^{(l,k_i)}(u_i') 2^{-\frac{\mu}{\lambda_i} d_i^{(l,i)}(x_i,u_i')}}$$

$$d_i^{(l,i)}(x_i,u_i) = \sum_{t,\hat{t},\boldsymbol{x}_{\setminus i},\boldsymbol{u}_{\setminus i}} d(t,\hat{t}) p_{T,\boldsymbol{X}}(t,\boldsymbol{x}) \Lambda^{(l,i)}(\hat{t}|u) \prod_{j<i} Q_j^{(l,*)}(u_j|x_j) \prod_{j'\geq i} Q_{j'}^{(l-1,*)}(u_{j'}|x_{j'})$$

Flowchart



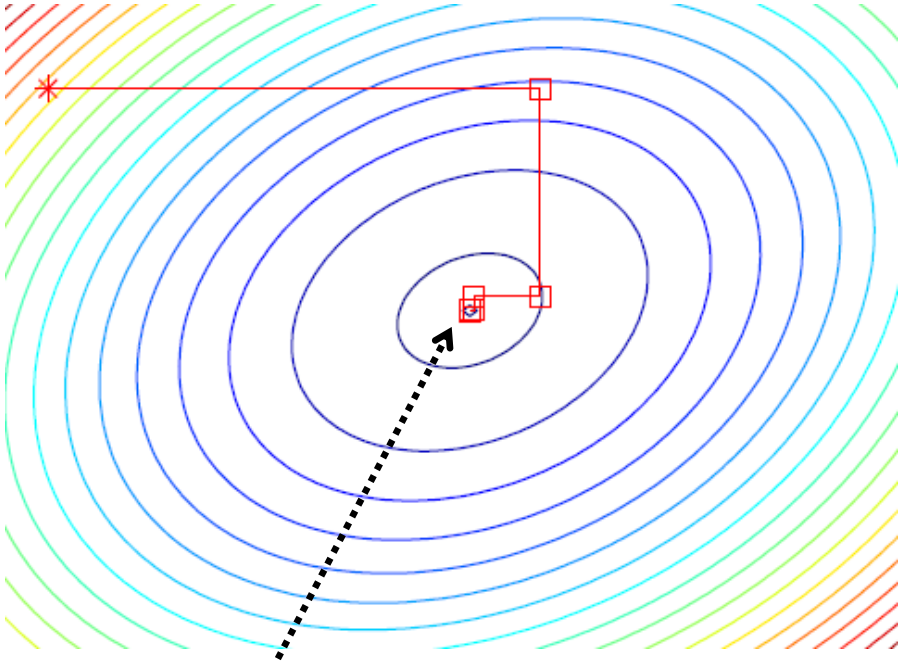
$$\Lambda^{(l',i')}(\hat{t}|\boldsymbol{u}) = \begin{cases} \alpha^{(l',i')}(\boldsymbol{u}) & \hat{t} \in \arg\min_{\hat{t}\in\hat{\mathcal{T}}} J^{(l,i)}(\hat{t},u) \\ 0 & otherwise \end{cases}$$

$$\sum_{\hat{t}} \Lambda^{(l',i')}(\hat{t}|\boldsymbol{u}) = 1$$

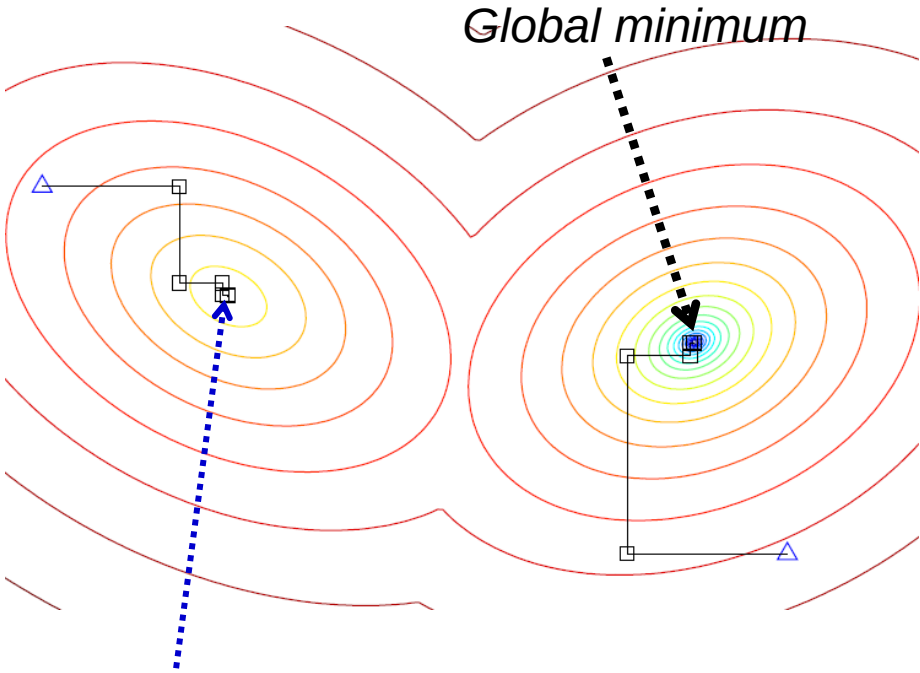
$$J^{(l,i)}(\hat{t},u) = \sum_{t,\hat{t},\boldsymbol{x}_{\setminus m},\boldsymbol{u}_{\setminus m}} d(t,\hat{t}) p_{T,\boldsymbol{X}}(t,\boldsymbol{x}) \prod_{j\le i} Q_j^{(l,*)}(u_j|x_j) \prod_{j'>i} Q_{j'}^{(l-1,*)}(u_{j'}|x_{j'})$$

Convergence

Convex & Non-Convex



Global minimum



Local minimum

Theorem

$\mathcal{L}_{\lambda\mu}(\boldsymbol{Q}, \boldsymbol{q}, \Lambda)$ is a convex function of $(\boldsymbol{Q}, \boldsymbol{q}, \Lambda)$

$\Rightarrow$ Ind. CEO BA will converge to a global minimum.

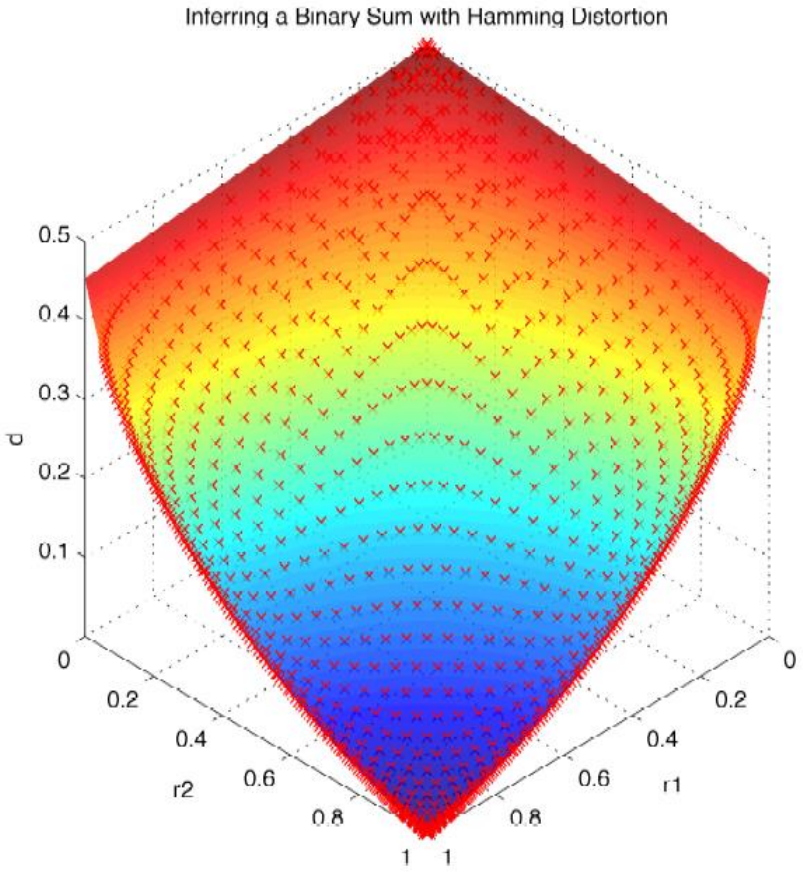
$\mathbb{E}\left[d(T, \hat{T})\right]$ is Convex

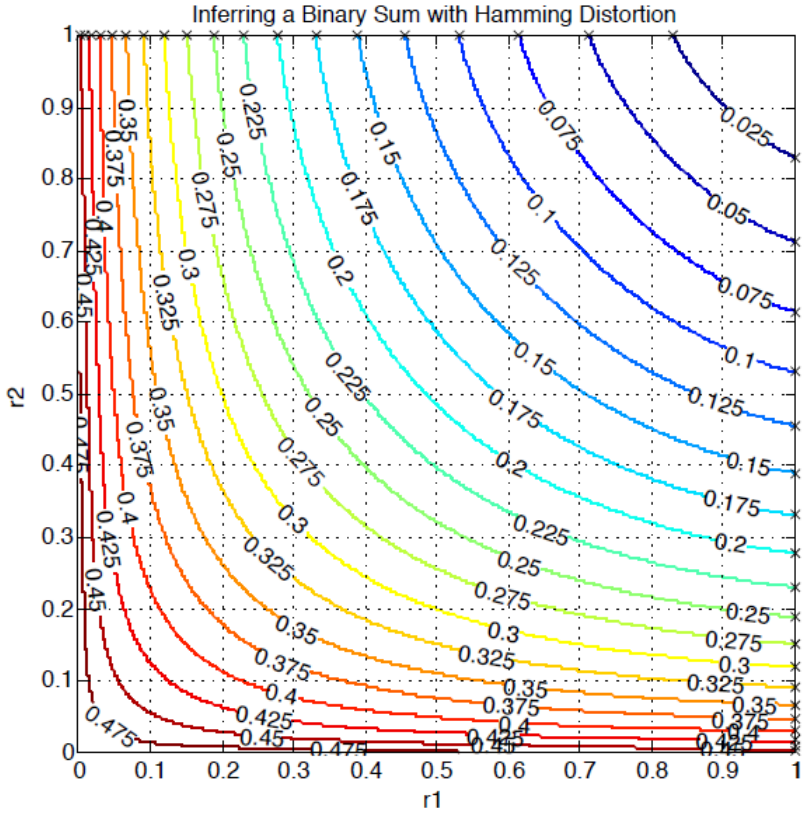
$\mathcal{L}_{\lambda\mu}(\boldsymbol{Q}, \boldsymbol{q}, \Lambda)$ is Convex

Global Convergence

Convex

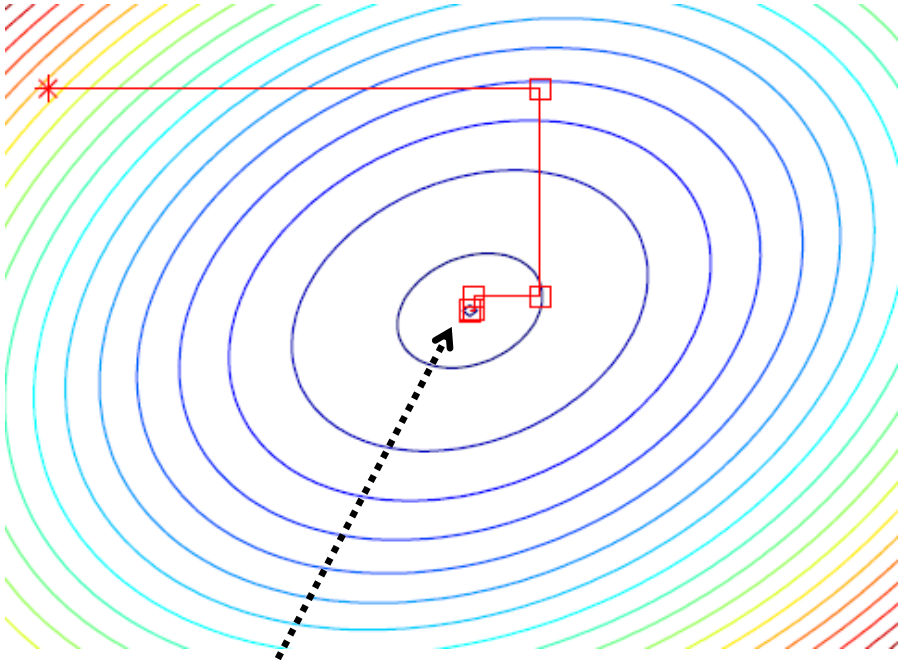
Binary Hamming, Global Convergence



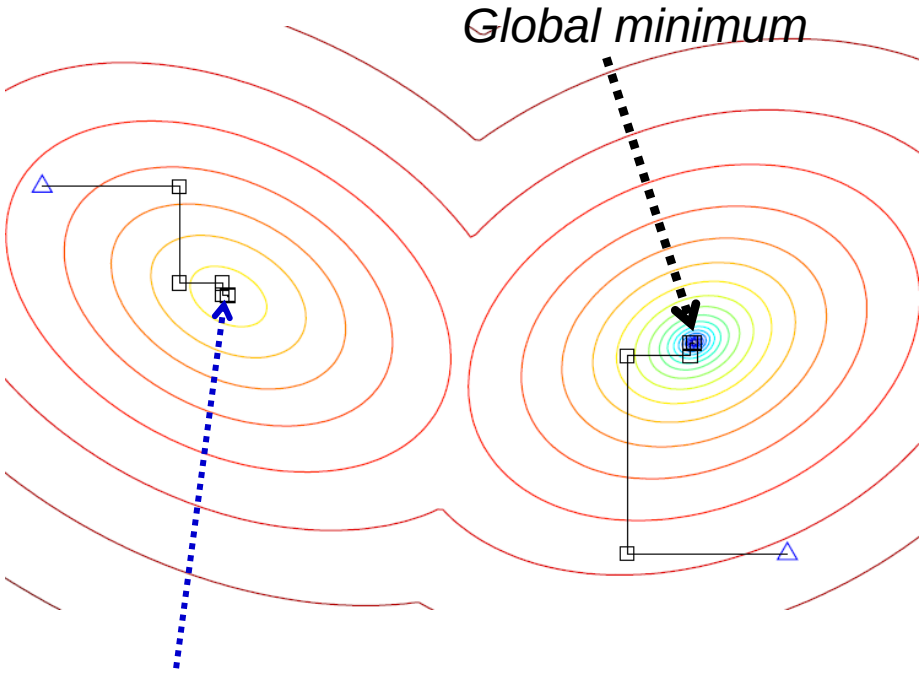


Convergence

Convex & Non-Convex



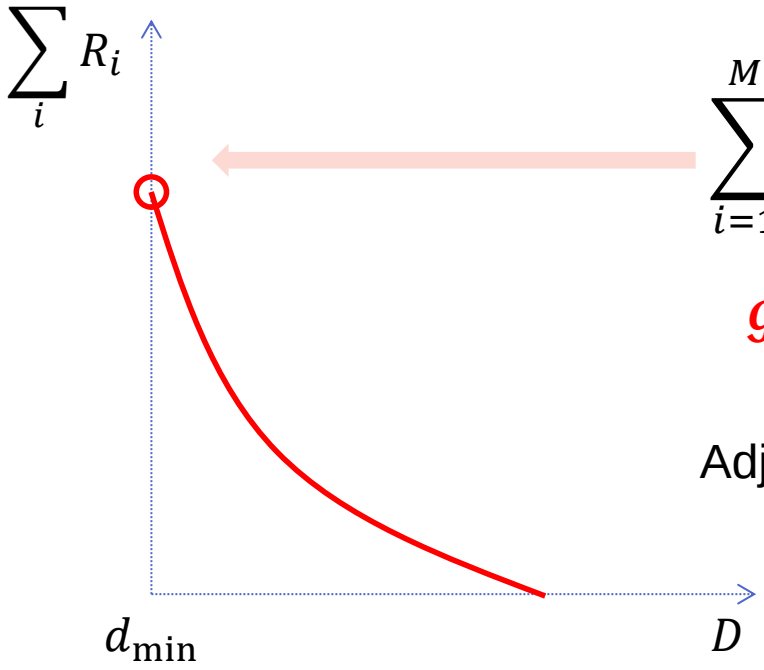
Global minimum



Local minimum

Non-convex

Initialization by Graph Entropy at $d_{\min}$



Convex optimization problem

$$\sum_{i=1}^{M} \min_{Q_i(\cdot|\cdot)\in\Gamma(\mathcal{G}_i)} I(X_i; U_i)$$

$\mathcal{G}_i$: *Characteristic graph* of vertices $\boldsymbol{\mathcal{X}}_i$

Adjacent x_i and x_i'

$$\exists\, \boldsymbol{x}_{\setminus i} \in \boldsymbol{X}_{\setminus i} \text{ s.t. } \hat{t}(x_i, \boldsymbol{x}_{\setminus i}) \neq \hat{t}(x_i', \boldsymbol{x}_{\setminus i})$$

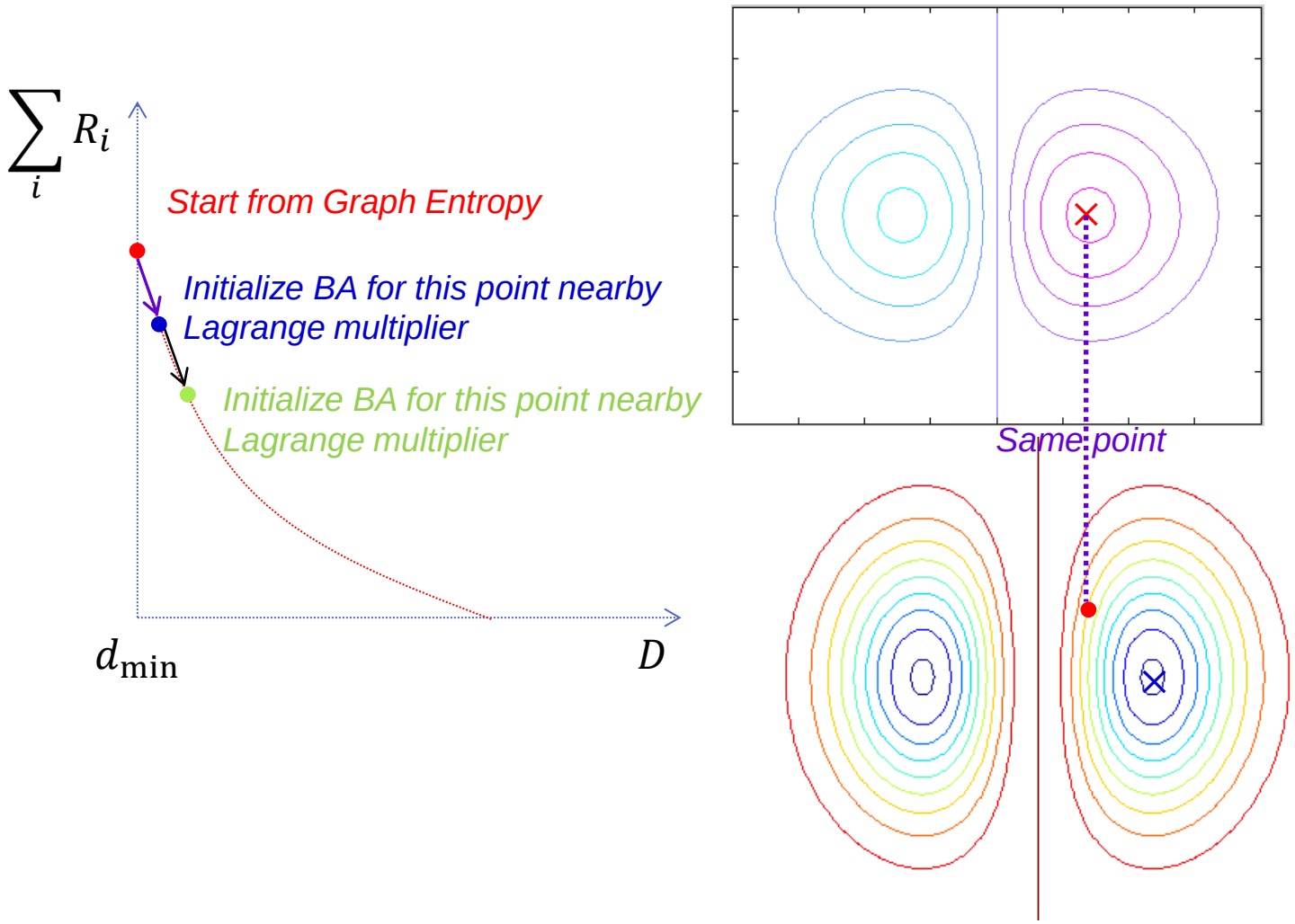
$\Gamma(\mathcal{G}_i)$: *Maximal independent sets* of $\mathcal{G}_i$

* V. Doshi, D. Shah, M. Medard, and M. Effros, "Functional compression through graph coloring," IEEE Transactions on Information Theory, vol. 56, no. 8, pp. 3901-3917, August 2010.

** V. Doshi, D. Shah, M. Medard, and S. Jaggi, "Graph coloring and conditional graph entropy," in 40th Asilomar Conf. on Signals, Systems, and Computers, 2006, pp. 2137{2141.

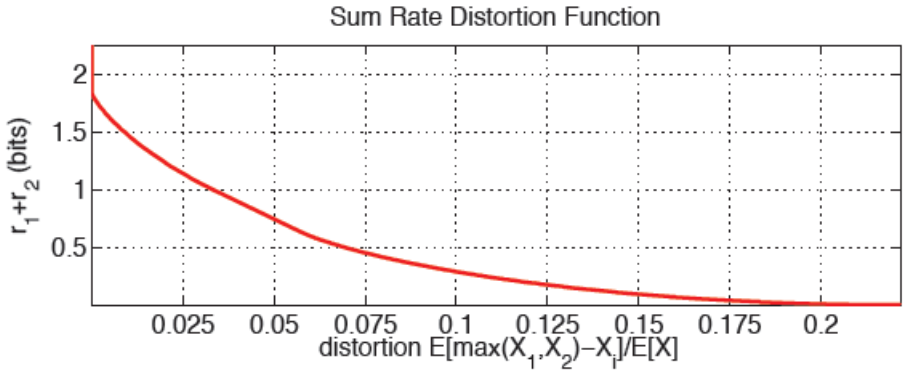
Non-convex (Continued)

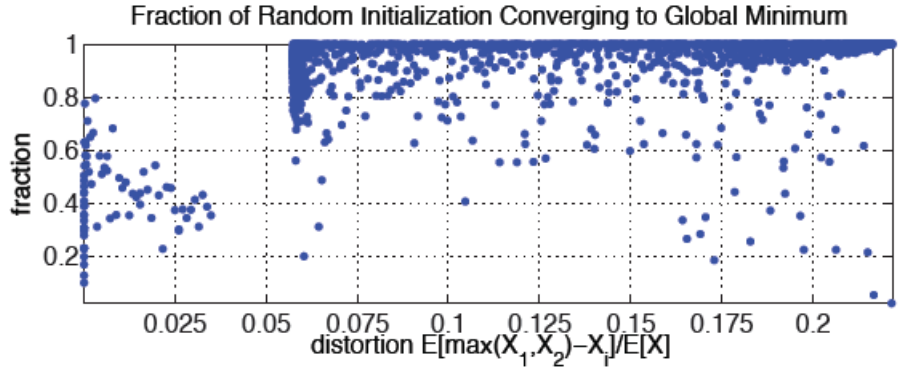
Tracking Previous Initialization,

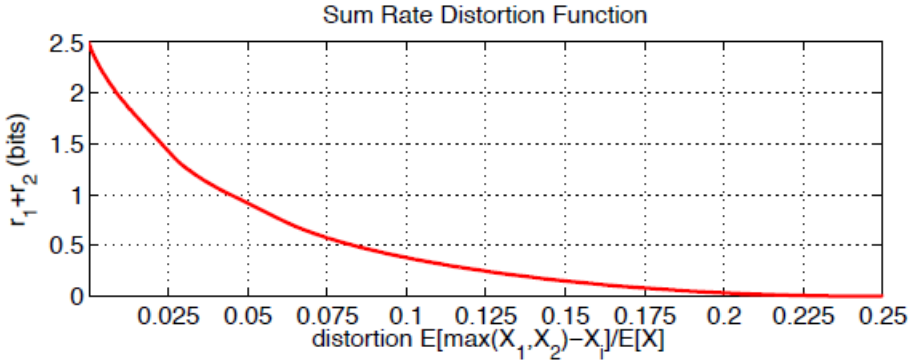


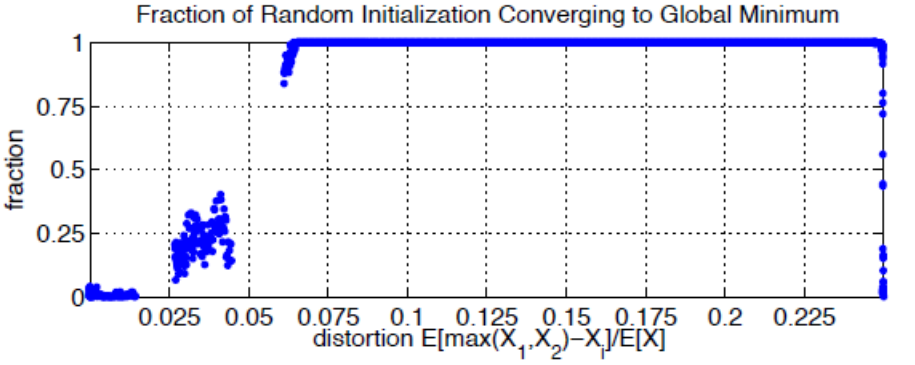
Non-convex Examples

Argmax [1 2 3] & [1 2 3 4]







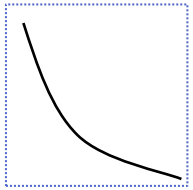


Calculating Convex Hulll

Redundancy Removal Description

Grid of $\vec{\lambda}$ and μ

different slope →



$$\min \sum_{i \in \mathcal{M}} \lambda_i I(X_i; U_i) + \mu \mathbb{E}\left[d(T, \hat{T})\right]$$

Redundancy Removal

$(\vec{r}^*, d^*)$

Polyhedral Rep. conversion

$$\left\{(\vec{r}, d) | \mathbf{H}\vec{r} + \vec{h}d \geq \mathbf{0}\right\}$$

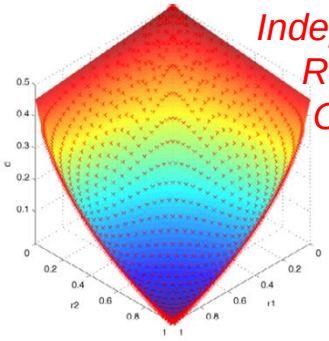
$$r_i^{(l)} = \sum_{(x_i, u_i)} p_{X_i}(x_i) Q_i^{(l,*)}(u_i|x_i) \log \frac{Q_i^{(l,*)}(u_i|x_i)}{q_i^{(l,*)}(u_i)}$$

$$d^{(l)} = \sum_{u,x,t,\hat{t}} d(t, \hat{t}) p_{T|\boldsymbol{X}}(t|\boldsymbol{x}) \Lambda^{(l,*)}(\hat{t}|\boldsymbol{u}) \prod_{i \in \mathcal{M}} Q_i^{(l,*)}(u_i|x_i)$$

$$\mathbf{H} = \begin{bmatrix} 0_{M+1 \times 1} & \mathbb{I}_{M+1} \\ 1_{K \times 1} & \mathbf{V} \end{bmatrix}$$

$$\mathbf{V} = \begin{bmatrix} r_1^{(1)} & \dots & r_M^{(1)} & d^{(1)} \\ r_1^{(2)} & \dots & r_M^{(2)} & d^{(2)} \\ \dots & \dots & \dots & \dots \end{bmatrix}$$

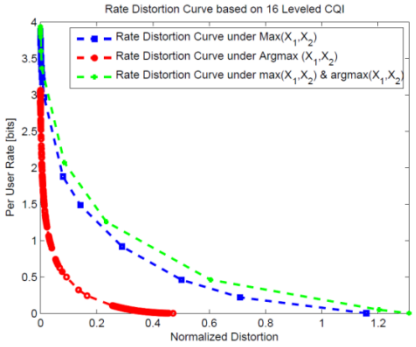
Overview



Independent CEO Rate Region Calculation

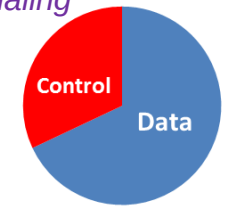
System Performance (Spectral Efficiency) for Data

Independent CEO with Resource Allocation

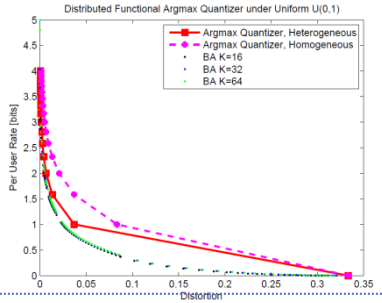


Information Theory Bound

LTE Control Signaling

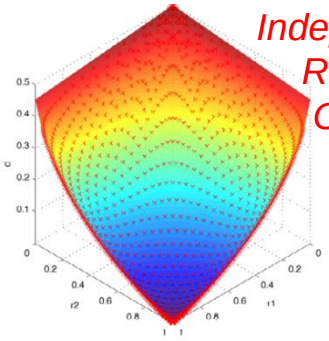


Practical Source Coding



Control (Collaboration) Info.

Overview

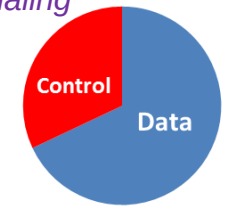


Independent CEO Rate Region Calculation

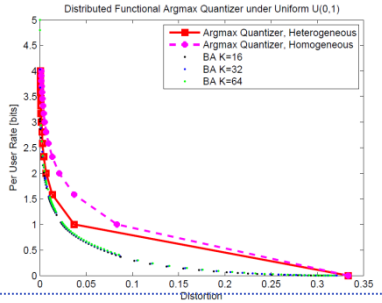
System Performance (Spectral Efficiency) for Data

Information Theory Bound

LTE Control Signaling

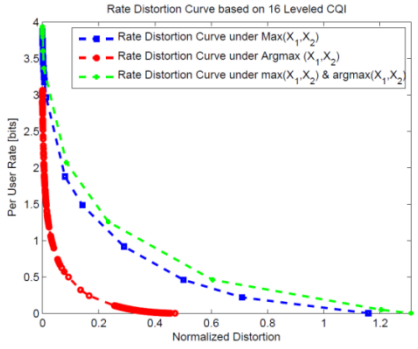


Practical Source Coding

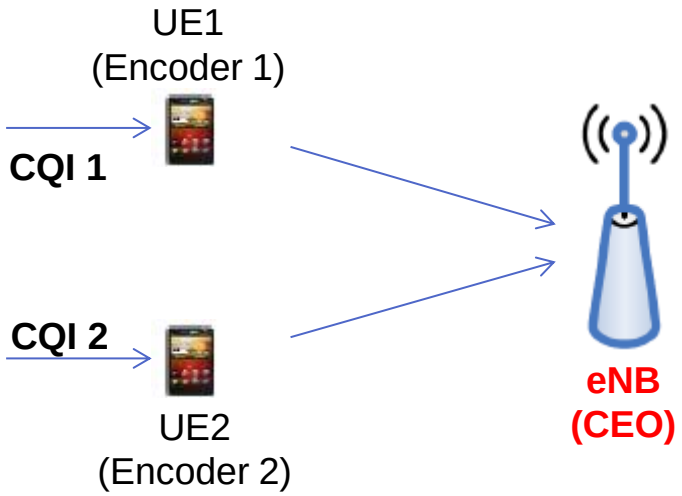


Control (Collaboration) Info.

Independent CEO with Resource Allocation



Overhead Performance Tradeoff*



Resource Allocation Function : Z

Inefficiency in Resource Allocation : D

Performance overhead tradeoff via *Rate Distortion Curve*

System Performance in **Spectral Efficiency**

Simple Model for Physical Layer employing DL communication

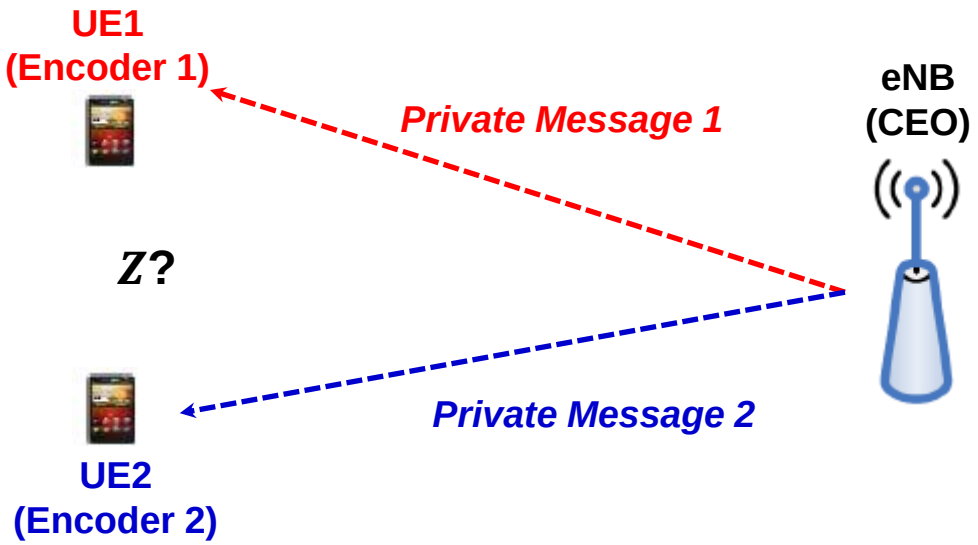
Broadcast Communication

Reteless coded Communication

Adaptive Modulation and Coding (AMC) Communication

*J. Ren, G. Ku, B. D. Boyle, S. Weber, and J. M. Walsh, "Overhead Performance Tradeoffs - A Resource Allocation Perspective," IEEE Transactions on Information Theory, to be submitted.

AMC Communication



Private messages

- Wish to select modulation and coding rate to match the channel capacity

  → Want to know a **UE** attaining maximum **CQI**

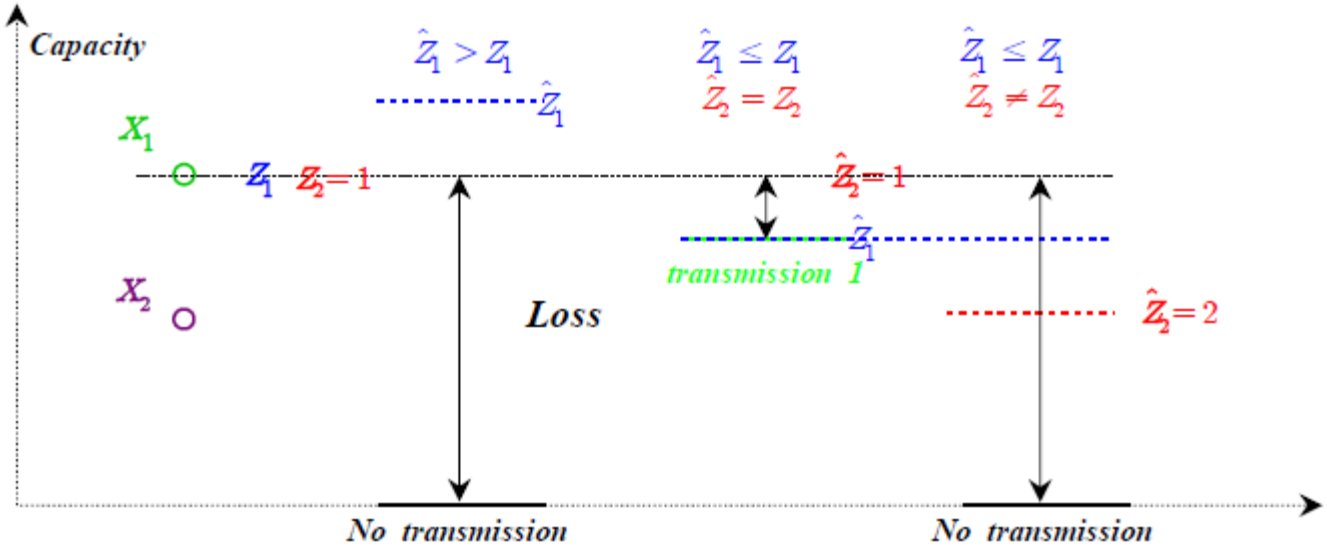
  → Want to know the **maximum CQI**

$$Z = (\max(X_1, \ldots, X_N), \operatorname{argmax}(X_1, \ldots, X_N))$$

Max & Argmax for AMC

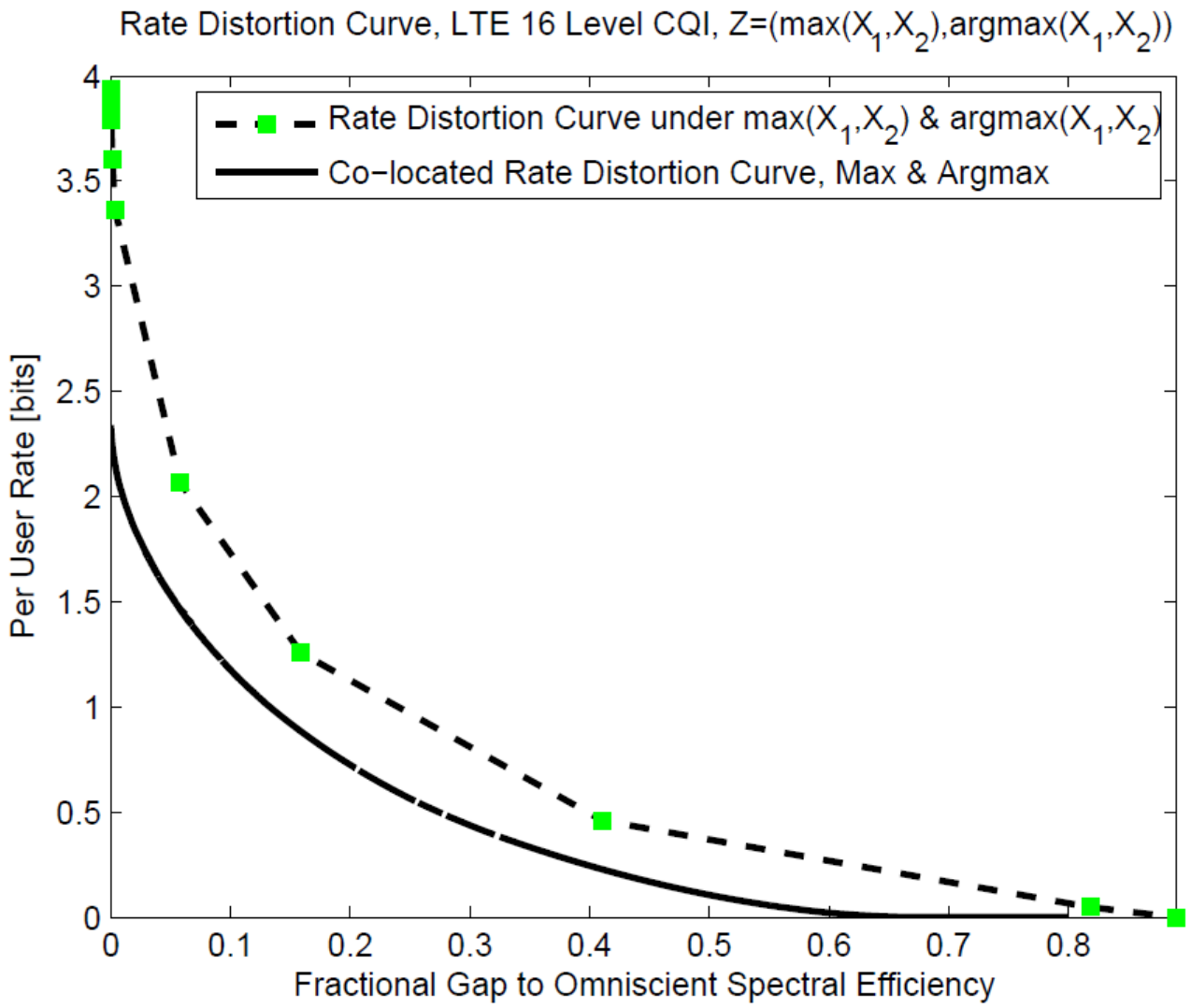
AMC $Z = (\max(X_1, X_2), \operatorname{argmax}(X_1, X_2))$

- **Estimation at Basestation**

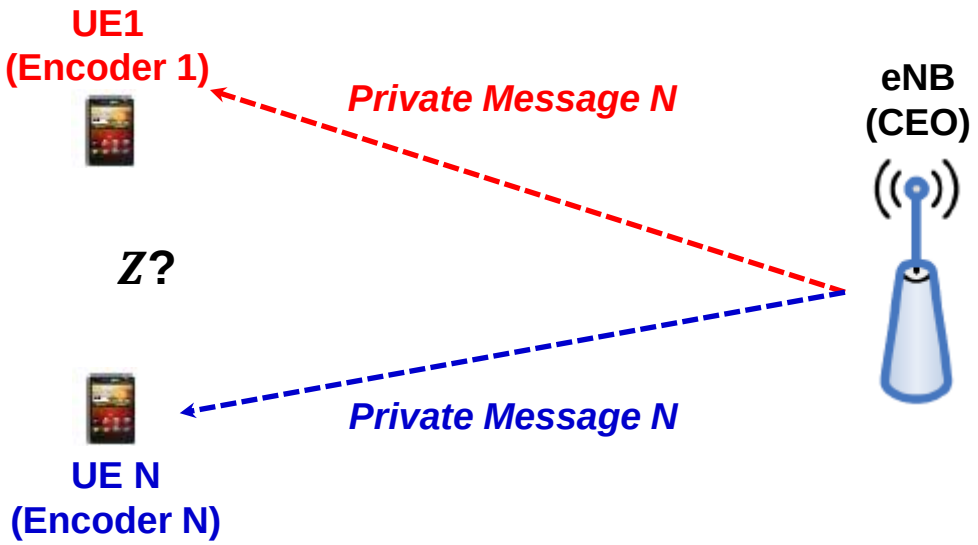


$$d\big((Z,i),(\hat{Z},\hat{\imath})\big) = \begin{cases} Z - \hat{Z} & \hat{Z} \le X_{\hat{\imath}} \\ Z & otherwise \end{cases}$$

Simulation : Max & Argmax



Rateless Coded Communication



Private messages

- Employs a **near-perfect rateless code** (Ex. HARQ)

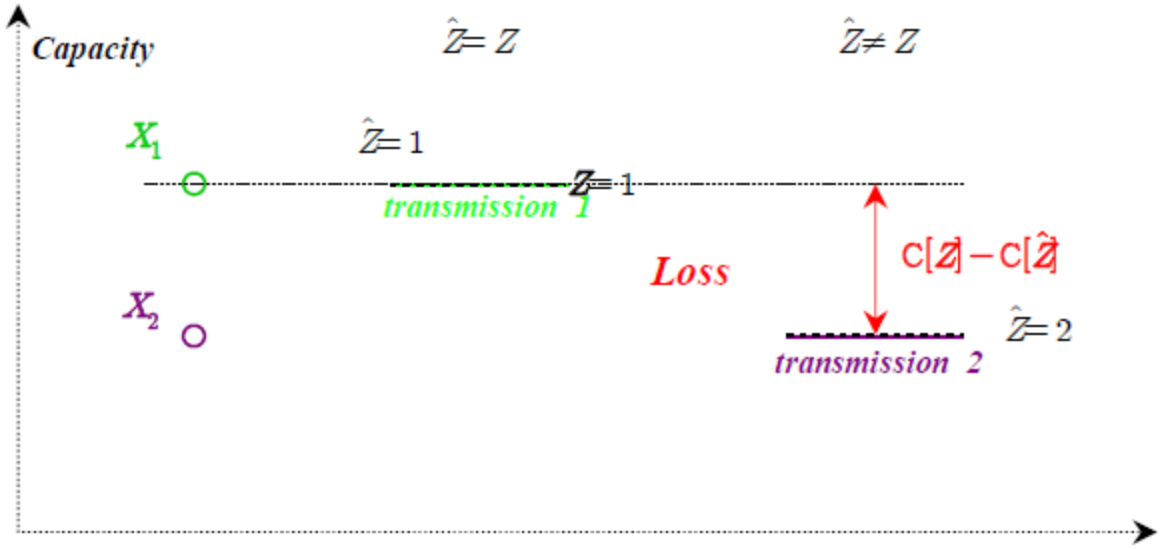
  → Need to know **a scheduled UE** attaining the **maximum CQI**

$$Z = \mathbf{argmax}(X_1, \ldots, X_N)$$

Argmax

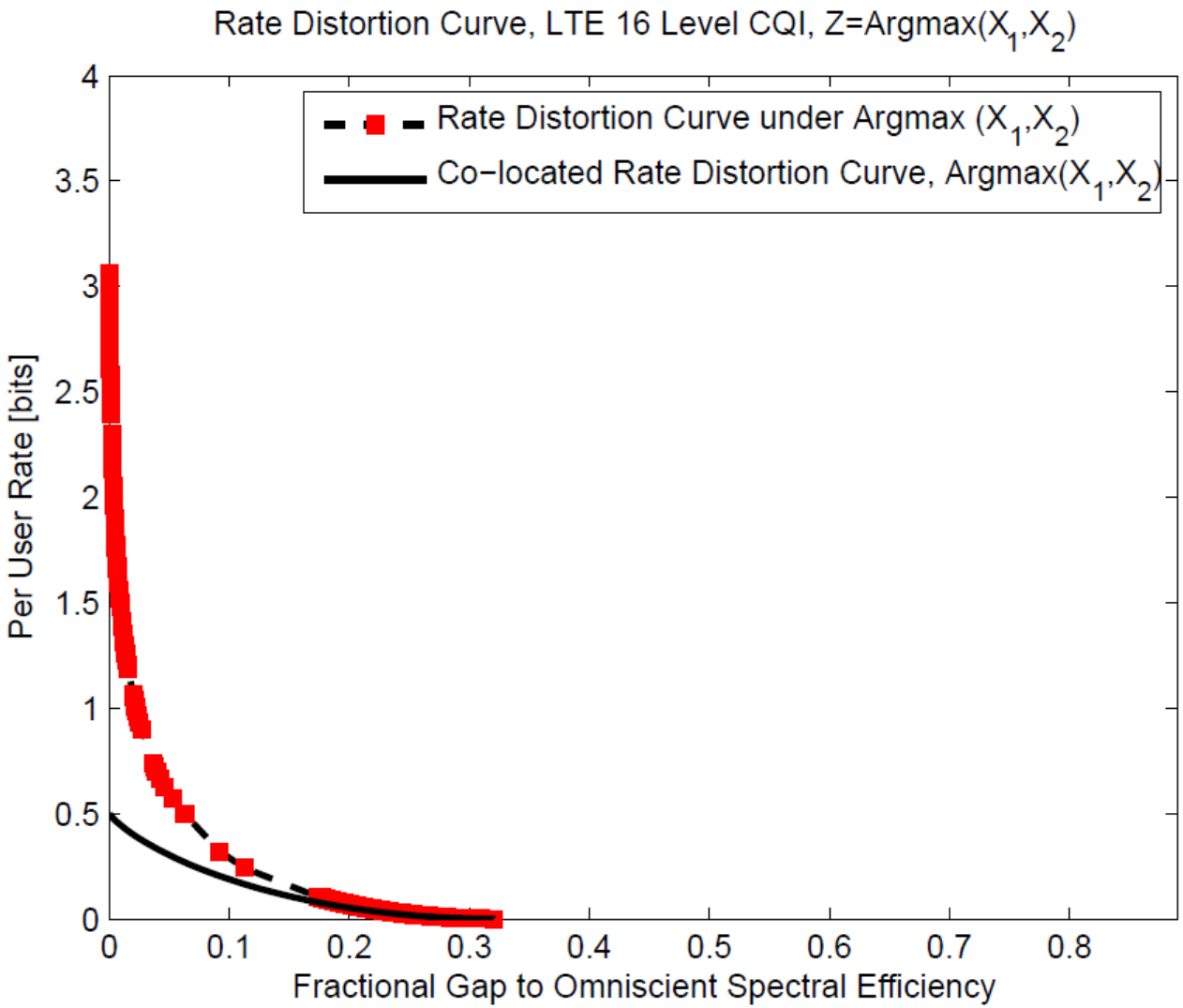
Rateless Communication $Z = \text{argmax}(X_1, X_2)$

- **Estimation at Basestation**

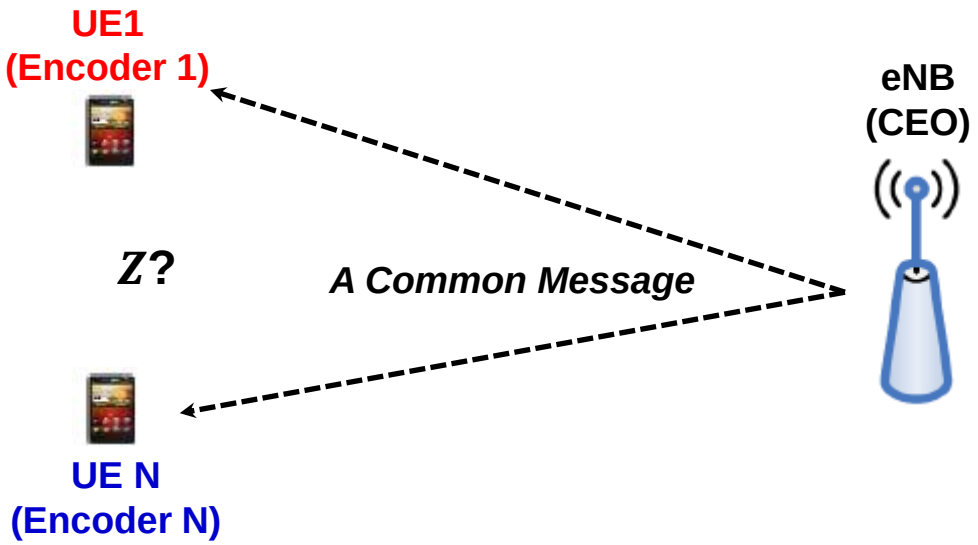


$$d(Z,\hat{Z}) = X_Z - X_{\hat{Z}}$$

Simulation : Argmax



Broadcast Communication



Common message

- For each UE to successfully decode on each channel

  → Need to know the **lowest CQI** level between UEs

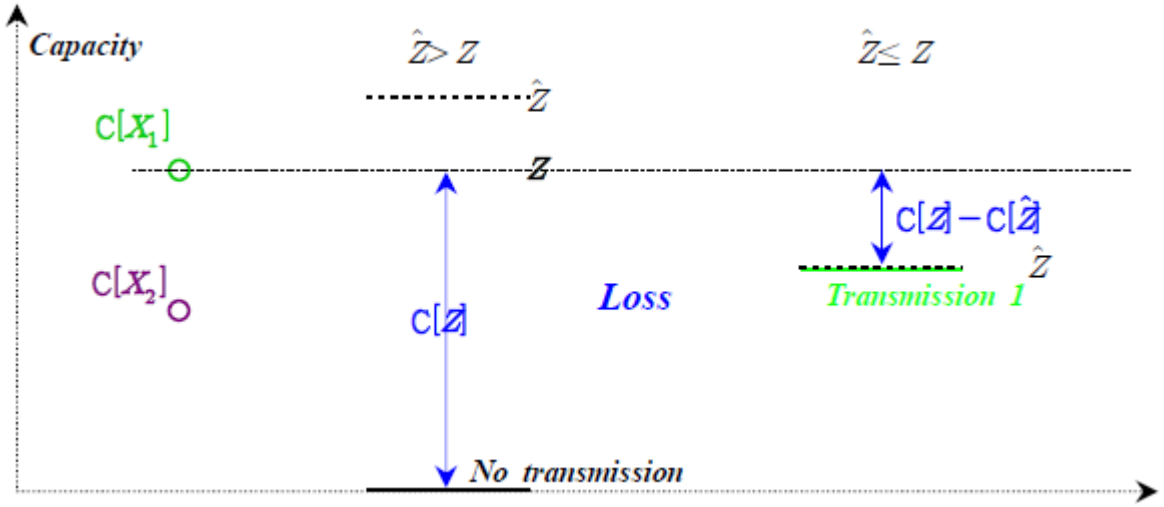
  → Want to know the **maximum negative CQI**

$$Z = \max(X_1, \ldots, X_N)$$

Max Function for Broadcast

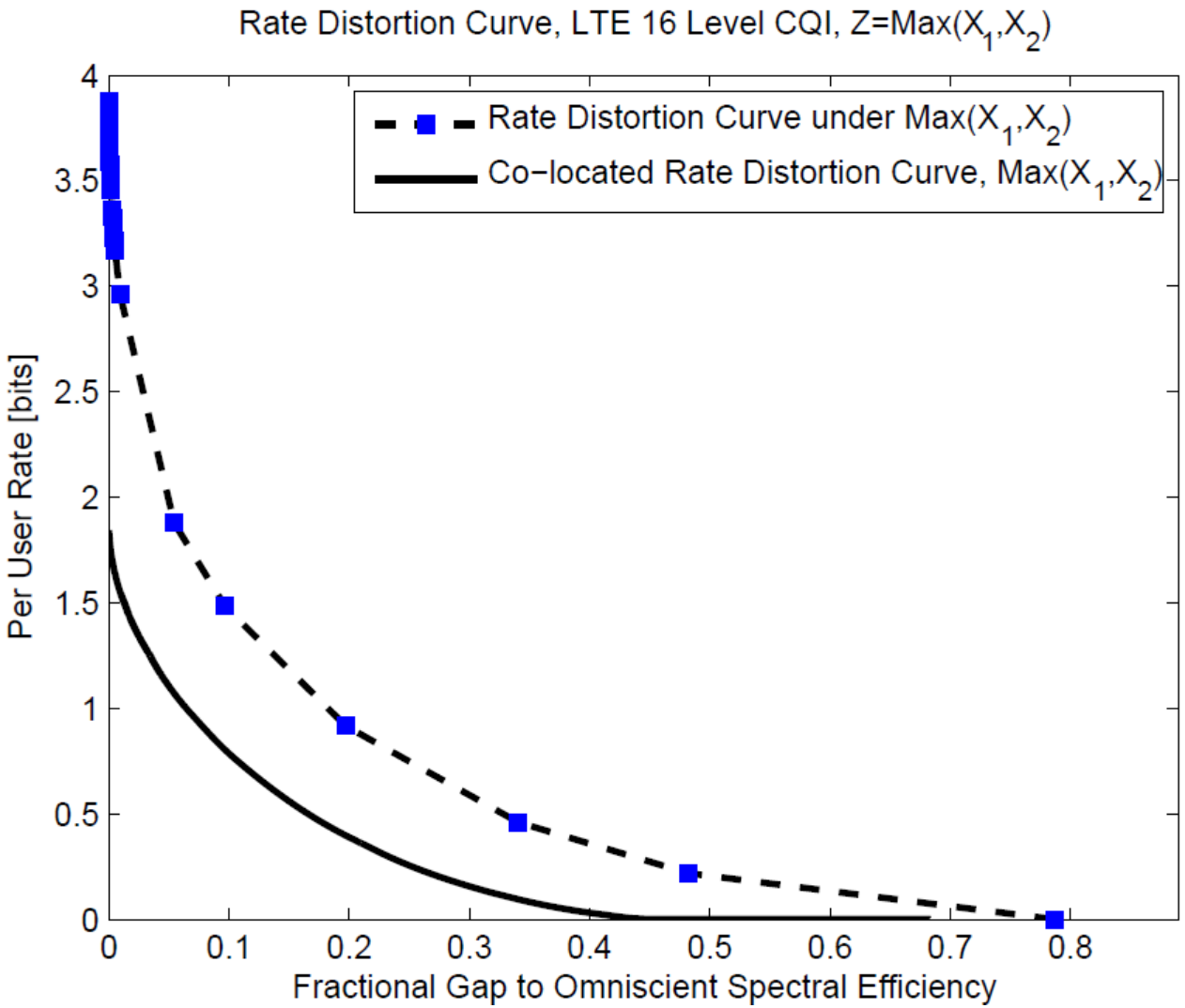
$$Z = \max(X_1, X_2)$$

- **Estimation at eNB**

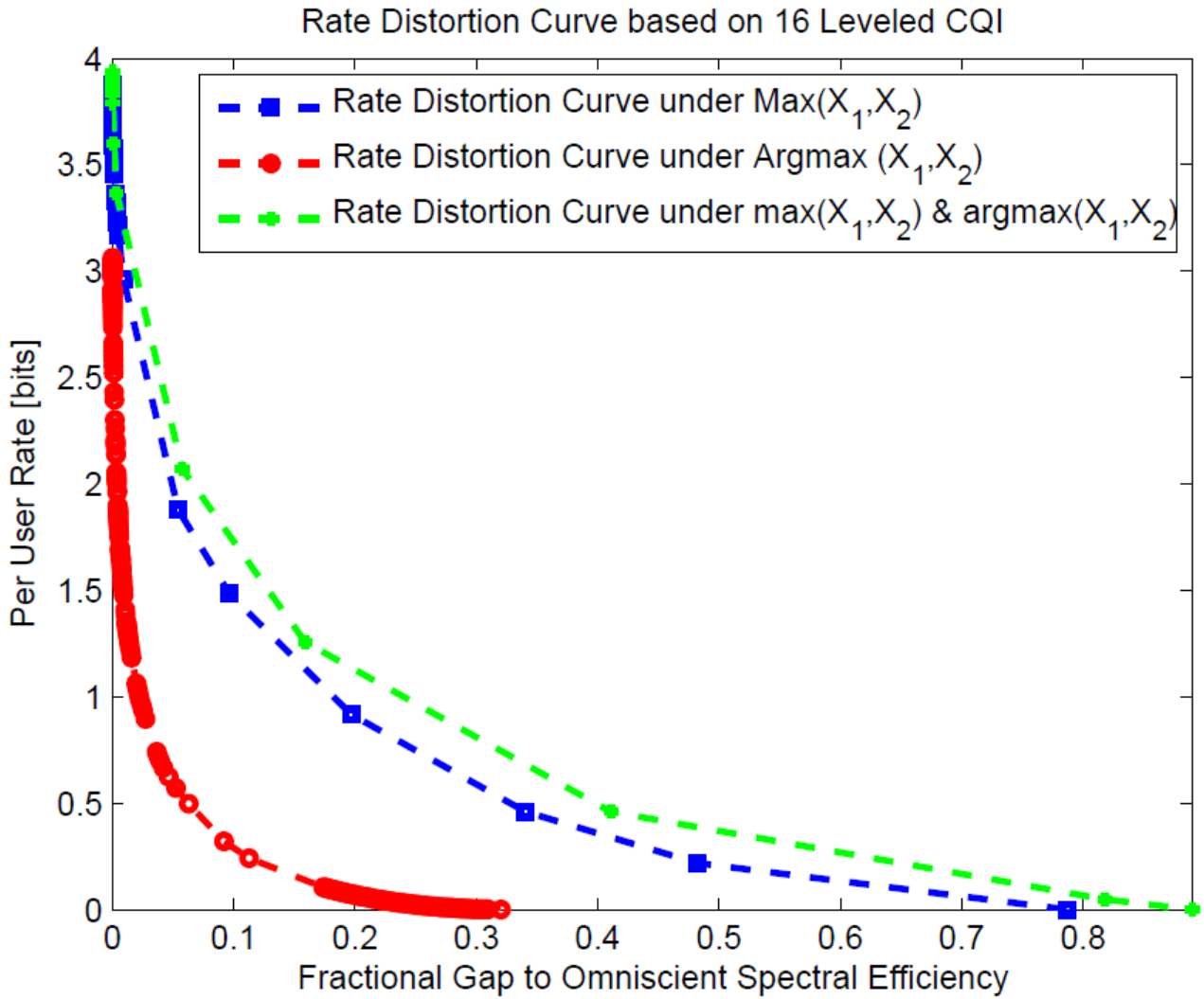


$$d(Z, \hat{Z}) = \begin{cases} Z - \hat{Z} & \hat{Z} \leq Z \\ Z & otherwise \end{cases}$$

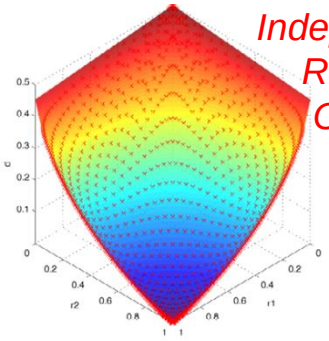
Simulation : Max



Simulation : Overall



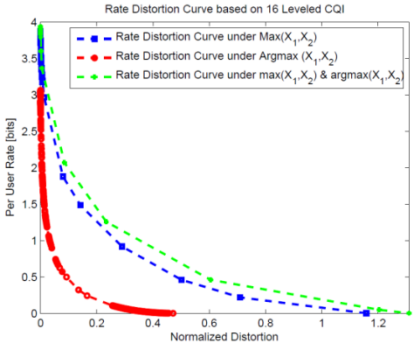
Overview



Independent CEO Rate Region Calculation

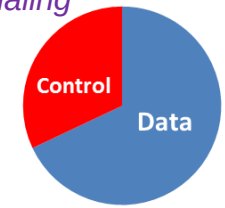
System Performance (Spectral Efficiency) for Data

Independent CEO with Resource Allocation

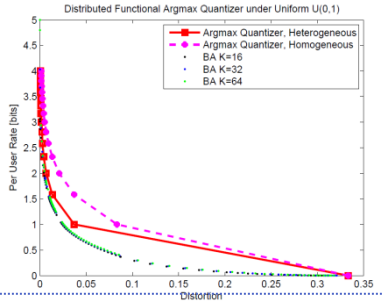


Information Theory Bound

LTE Control Signaling

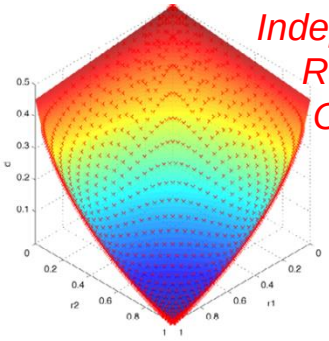


Practical Source Coding



Control (Collaboration) Info.

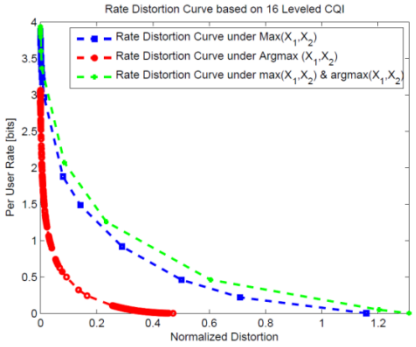
Overview



Independent CEO Rate Region Calculation

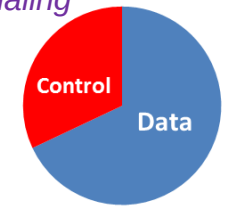
System Performance (Spectral Efficiency) for Data

Independent CEO with Resource Allocation

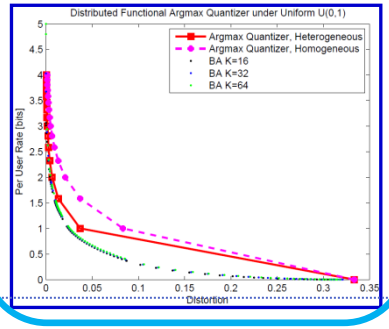


Information Theory Bound

LTE Control Signaling



Practical Source Coding



Control (Collaboration) Info.

Quantizer

Flowchart

Find Bayes detector for fixed thresholds

$$\arg\min_{\hat{t}} \mathbb{E}\left[d(T,\hat{t}) | X_1 \in \mathbf{I}_{1,k_1}, \ldots, X_M \in \mathbf{I}_{1,k_M}\right]$$

Average distortion of Bayes detector in terms of thresholds

$$\min \mathbb{E}\left[d(T,\hat{T})\right]$$

Minimize distortion w.r.t. thresholds

$$\min \mathbb{E}\left[d(T,\hat{T})\right]$$

Max & Argmax Quantizer

Find Bayes detector for fixed thresholds

$$\arg\min_{\hat{t}} \mathbb{E}\left[d(T,\hat{t})|X_1 \in \mathbf{I}_{1,k_1},\ldots,X_M \in \mathbf{I}_{1,k_M}\right]$$

$$\hat{t}^* = \begin{cases} \hat{t}_1^* & \text{If } l_{1,k_1-1} \geq l_{2,k_2} \\ \hat{t}_2^* & \text{If } l_{2,k_2-1} \geq l_{1,k_1} \\ \hat{t}_0^* & \text{If } \max(l_{2,k_2-1}, l_{1,k_1-1}) \leq \min(l_{1,k_1}, l_{2,k_2}) \leq \max(l_{1,k_1}, l_{2,k_2}) \end{cases}$$

Average distortion of Bayes detector in terms of thresholds

$$\min \mathbb{E}\left[d(T,\hat{T})\right]$$

Minimize distortion w.r.t. thresholds

$$\min \mathbb{E}\left[d(T,\hat{T})\right]$$

$$\begin{aligned}
&\mathbb{E}\left[d(T,\hat{t})\right] \\
&= \sum_{(k_1,k_2)\in\mathbf{T}_1} \left\{ \int_{l_{1,k_1-1}}^{l_{1,k_1}} x_1 f_{X_1}(x_1)dx_1 - \hat{t}_1^* \left[F_{X_1}(l_{1,k_1}) - F_{X_1}(\hat{t}_1^*)\right] \right\} \left[F_{X_2}(l_{2,k_2}) - F_{X_2}(l_{2,k_2-1})\right] \\
&+ \sum_{(k_1,k_2)\in\mathbf{T}_2} \left\{ \int_{l_{2,k_2-1}}^{l_{2,k_2}} x_2 f_{X_2}(x_2)dx_2 - \hat{t}_2^* \left[F_{X_2}(l_{2,k_2}) - F_{X_2}(\hat{t}_2^*)\right] \right\} \left[F_{X_1}(l_{1,k_1}) - F_{X_1}(l_{1,k_1-1})\right] \\
&+ \sum_{(k_1,k_2)\in\mathbf{T}_{01}} \int_{\max(l_{1,k_1-1},l_{2,k_2-1})}^{l_{2,k_2}} t\left[f_{X_1}(t)[F_{X_2}(t) - F_{X_2}(l_{2,k_2-1})] + [F_{X_1}(t) - F_{X_1}(l_{1,k_1-1})]f_{X_2}(t)\right]dt \\
&+ \sum_{(k_1,k_2)\in\mathbf{T}_{01}} \int_{l_{2,k_2}}^{l_{1,k_1}} t f_{X_1}(t)dt(F_{X_2}(l_{2,k_2}) - F_{X_2}(l_{2,k_2-1}) + \sum_{(k_1,k_2)\in\mathbf{Z}_{01}} \max_{\hat{t}_1^*,\hat{t}_2^*}\left[w(\hat{t}_1^*), w(\hat{t}_2^*)\right] \\
&+ \sum_{(k_1,k_2)\in\mathbf{T}_{02}} \int_{\max(l_{1,k_1-1},l_{2,k_2-1})}^{l_{1,k_1}} t\left[f_{X_2}(t)[F_{X_1}(t) - F_{X_1}(l_{1,k_1-1})] + [F_{X_2}(t) - F_{X_2}(l_{2,k_2-1})]f_{X_1}(t)\right]dt \\
&+ \sum_{(k_1,k_2)\in\mathbf{T}_{02}} \int_{l_{1,k_1}}^{l_{2,k_2}} t f_{X_2}(t)dt(F_{X_1}(l_{1,k_1}) - F_{X_1}(l_{1,k_1-1}) + \sum_{(k_1,k_2)\in\mathbf{Z}_{02}} \max_{\hat{t}_1^*,\hat{z}_2^*}\left[w(\hat{z}_1^*), w(\hat{z}_2^*)\right]
\end{aligned}$$

Max & Argmax Quantizer

Find Bayes detector for fixed thresholds

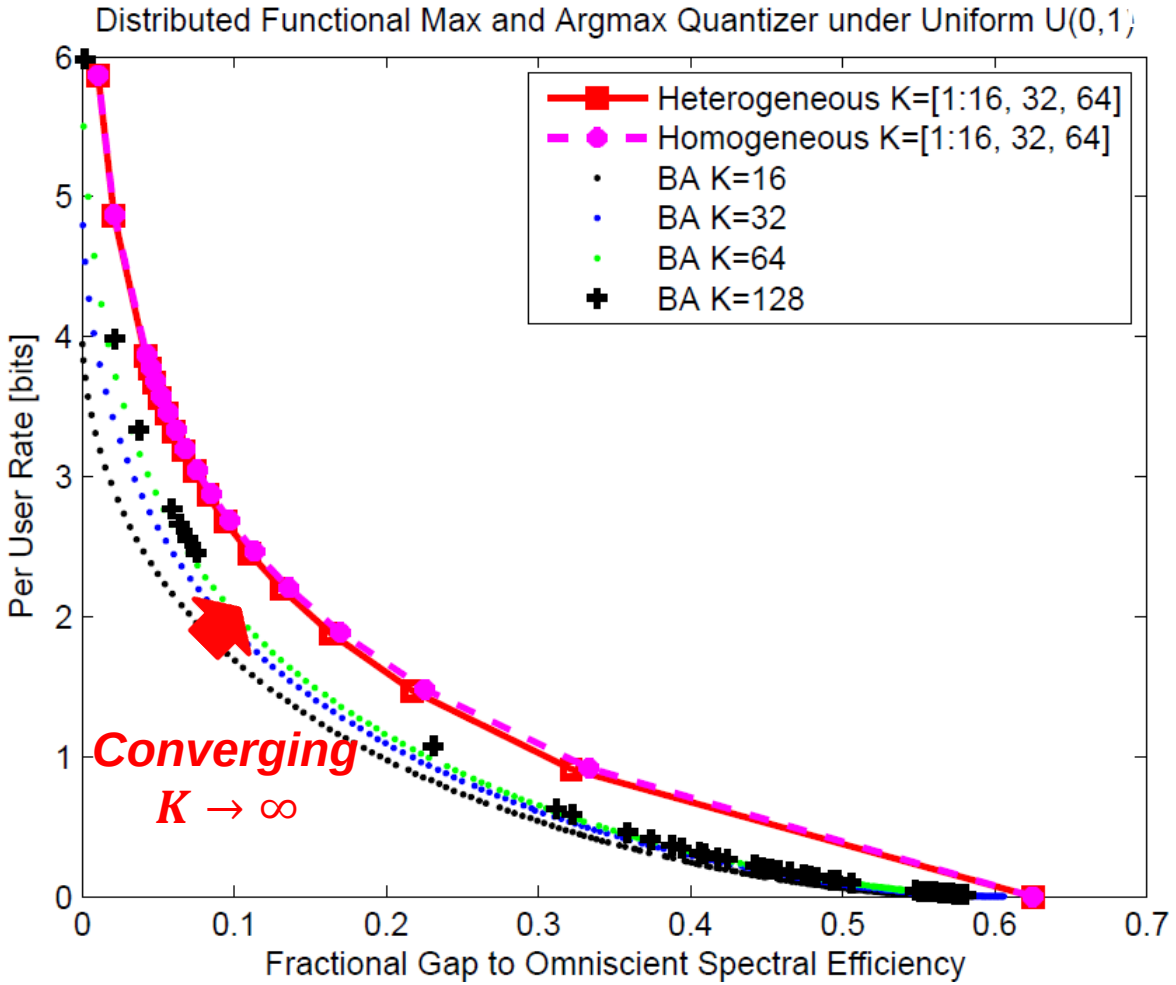
$$\arg\min_{\hat{t}} \mathbb{E}\left[d(T,\hat{t}) | X_1 \in \mathrm{I}_{1,k_1}, \ldots, X_M \in \mathrm{I}_{1,k_M}\right]$$

Average distortion of Bayes detector in terms of thresholds

$$\min \mathbb{E}\left[d(T,\hat{T})\right]$$

Minimize distortion w.r.t. thresholds

$$\min \mathbb{E}\left[d(T,\hat{T})\right]$$



Argmax Quantizer

Find Bayes detector for fixed thresholds

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$$\hat{t}^* = \begin{cases} 1 & \text{If } l_{1,k_1-1} \geq l_{2,k_2} \\ 2 & \text{If } l_{2,k_2-1} \geq l_{1,k_1} \\ \hat{t}_0^* & \text{If } \max(l_{2,k_2-1}, l_{1,k_1-1}) \leq \min(l_{1,k_1}, l_{2,k_2}) \leq \max(l_{1,k_1}, l_{2,k_2}) \end{cases}$$

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&= \sum_{(k_1,k_2)\in\mathbf{T}_{01}} \int_{\max(l_{1,k_1-1},l_{2,k_2-1})}^{l_{2,k_2}} t\left[f_{X_1}(t)[F_{X_2}(t)-F_{X_2}(l_{2,k_2-1})]+[F_{X_1}(t)-F_{X_1}(l_{1,k_1-1})]f_{X_2}(t)\right]dt \\
&+ \sum_{(k_1,k_2)\in\mathbf{T}_{01}} \int_{l_{2,k_2}}^{l_{1,k_1}} t f_{X_1}(z)dt(F_{X_2}(l_{2,k_2})-F_{X_2}(l_{2,k_2-1}) \\
&+ \sum_{(k_1,k_2)\in\mathbf{T}_{01}} \max\left(\mathbb{E}\left[X_1|X_1\in\mathbf{I}_{1,k_1}\right],\mathbb{E}\left[X_2|X_2\in\mathbf{I}_{2,k_2}\right]\right) \\
&+ \sum_{(k_1,k_2)\in\mathbf{T}_{02}} \int_{\max(l_{1,k_1-1},l_{2,k_2-1})}^{l_{1,k_1}} t\left[f_{X_2}(t)[F_{X_1}(t)-F_{X_1}(l_{1,k_1-1})]+[F_{X_2}(t)-F_{X_2}(l_{2,k_2-1})]f_{X_1}(t)\right]dt \\
&+ \sum_{(k_1,k_2)\in\mathbf{T}_{02}} \int_{l_{1,k_1}}^{l_{2,k_2}} t f_{X_2}(t)dt(F_{X_1}(l_{1,k_1})-F_{X_1}(l_{1,k_1-1}) \\
&+ \sum_{(k_1,k_2)\in\mathbf{T}_{02}} \max\left(\mathbb{E}\left[X_1|X_1\in\mathbf{I}_{1,k_1}\right],\mathbb{E}\left[X_2|X_2\in\mathbf{I}_{2,k_2}\right]\right)
\end{aligned}$$

Minimize distortion w.r.t. thresholds

$$\min \mathbb{E}\left[d(T,\hat{T})\right]$$

Argmax Quantizer

Find Bayes detector for fixed thresholds

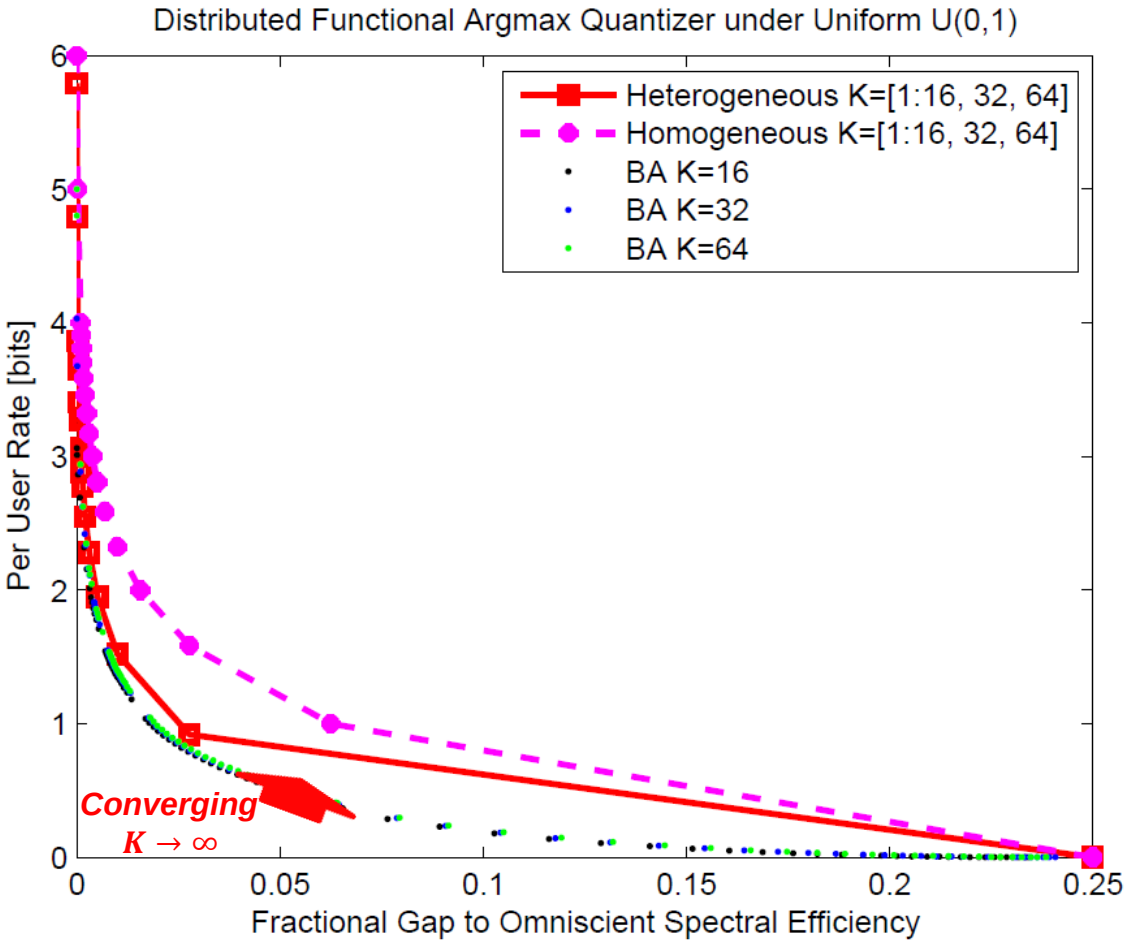
$$\arg\min_{\hat{t}} \mathbb{E}\left[d(T,\hat{t}) | X_1 \in \mathrm{I}_{1,k_1}, \ldots, X_M \in \mathrm{I}_{1,k_M}\right]$$

Average distortion of Bayes detector in terms of thresholds

$$\min \mathbb{E}\left[d(T,\hat{T})\right]$$

Minimize distortion w.r.t. thresholds

$$\min \mathbb{E}\left[d(T,\hat{T})\right]$$



Max Quantizer

Find Bayes detector for fixed thresholds

$$\arg\min_{\hat{t}} \mathbb{E}\left[d(T,\hat{t}) | X_1 \in \mathbf{I}_{1,k_1}, \ldots, X_M \in \mathbf{I}_{1,k_M}\right]$$

$$\hat{t}^* = \begin{cases} \hat{t}_1^* & \text{If } l_{1,k_1-1} \geq l_{2,k_2} \\ \hat{t}_2^* & \text{If } l_{2,k_2-1} \geq l_{1,k_1} \\ \hat{t}_{01}^* = \arg\max_{\hat{t}_{11}^*, \hat{t}_{12}^*} \left[w_{11}(\hat{t}_{11}^*), w_{12}(\hat{t}_{12}^*)\right] & \text{If } \max(l_{2,k_2-1}, l_{1,k_1-1}) < l_{2,k_2} \leq l_{1,k_1} \\ \hat{t}_{02}^* = \arg\max_{\hat{t}_{21}^*, \hat{t}_{22}^*} \left[w_{21}(\hat{t}_{21}^*), w_{22}(\hat{t}_{22}^*)\right] & \text{If } \max(l_{1,k_1-1}, l_{2,k_2-1}) < l_{1,k_1} \leq l_{2,k_2} \end{cases}$$

Average distortion of Bayes detector in terms of thresholds

$$\min \mathbb{E}\left[d(T,\hat{T})\right]$$

$$\begin{aligned} &\mathbb{E}\left[d(T,\hat{t})\right] \\ &= \sum_{(k_1,k_2)\in\mathbf{T}_1} \left\{ \int_{l_{1,k_1-1}}^{l_{1,k_1}} x_1 f_{X_1}(x_1) dx_1 - \hat{t}_1^* \left[F_{X_1}(l_{1,k_1}) - F_{X_1}(\hat{t}_1^*)\right] \right\} \left[F_{X_2}(l_{2,k_2}) - F_{X_2}(l_{2,k_2-1})\right] \\ &+ \sum_{(k_1,k_2)\in\mathbf{T}_2} \left\{ \int_{l_{2,k_2-1}}^{l_{2,k_2}} x_2 f_{X_2}(x_2) dx_2 - \hat{t}_2^* \left[F_{X_2}(l_{2,k_2}) - F_{X_2}(\hat{t}_2^*)\right] \right\} \left[F_{X_1}(l_{1,k_1}) - F_{X_1}(l_{1,k_1-1})\right] \\ &+ \sum_{(k_1,k_2)\in\mathbf{T}_{01}} \int_{\max(l_{1,k_1-1}, l_{2,k_2-1})}^{l_{2,k_2}} t\left[f_{X_1}(t)[F_{X_2}(t) - F_{X_2}(l_{2,k_2-1})] + [F_{X_1}(t) - F_{X_1}(l_{1,k_1-1})] f_{X_2}(t)\right] dt \\ &+ \sum_{(k_1,k_2)\in\mathbf{T}_{01}} \int_{l_{2,k_2}}^{l_{1,k_1}} t f_{X_1}(t) dt \left(F_{X_2}(l_{2,k_2}) - F_{X_2}(l_{2,k_2-1}) + \sum_{(k_1,k_2)\in\mathbf{T}_{01}} \max_{\hat{t}_{11}^*, \hat{t}_{12}^*} \left[w_{11}(\hat{t}_{11}^*), w_{12}(\hat{t}_{12}^*)\right]\right. \\ &+ \sum_{(k_1,k_2)\in\mathbf{T}_{02}} \int_{\max(l_{1,k_1-1}, l_{2,k_2-1})}^{l_{1,k_1}} t\left[f_{X_2}(t)[F_{X_1}(t) - F_{X_1}(l_{1,k_1-1})] + [F_{X_2}(t) - F_{X_2}(l_{2,k_2-1})] f_{X_1}(t)\right] dt \\ &+ \sum_{(k_1,k_2)\in\mathbf{T}_{02}} \int_{l_{1,k_1}}^{l_{2,k_2}} t f_{X_2}(t) dt \left(F_{X_1}(l_{1,k_1}) - F_{X_1}(l_{1,k_1-1}) + \sum_{(k_1,k_2)\in\mathbf{T}_{02}} \max_{\hat{t}_{21}^*, \hat{t}_{22}^*} \left[w_{21}(\hat{t}_{21}^*), w_{22}(\hat{t}_{22}^*)\right]\right. \end{aligned}$$

Minimize distortion w.r.t. thresholds

$$\min \mathbb{E}\left[d(T,\hat{T})\right]$$

Max Quantizer

Find Bayes detector for fixed thresholds

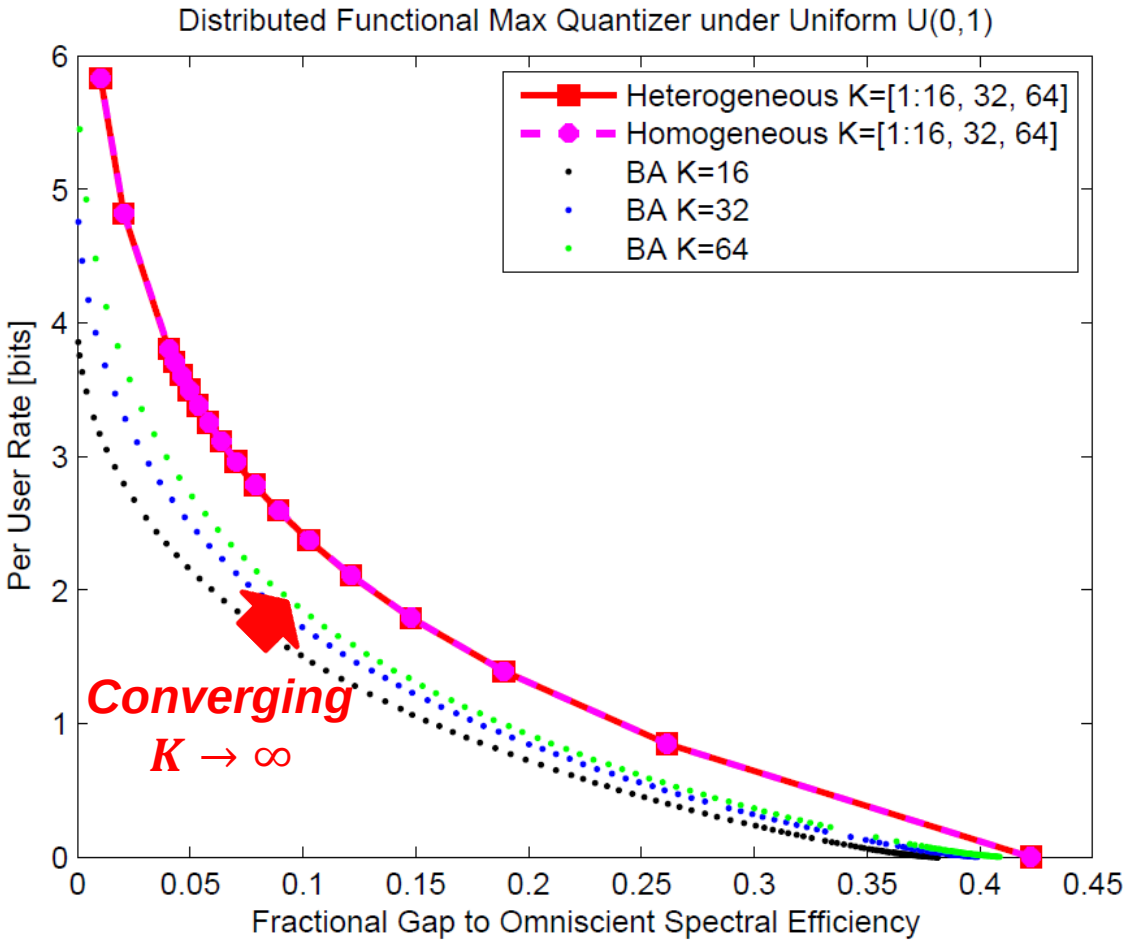
$$\arg\min_{\hat{t}} \mathbb{E}\left[d(T,\hat{t}) | X_1 \in \mathrm{I}_{1,k_1}, \ldots, X_M \in \mathrm{I}_{1,k_M}\right]$$

Average distortion of Bayes detector in terms of thresholds

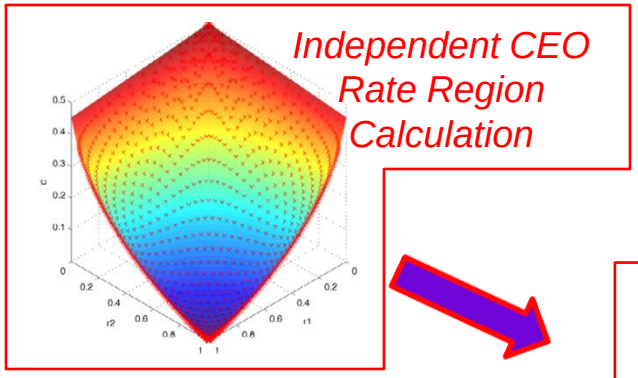
$$\min \mathbb{E}\left[d(T,\hat{T})\right]$$

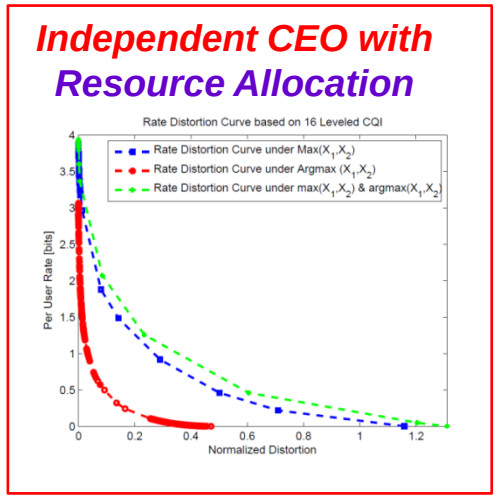
Minimize distortion w.r.t. thresholds

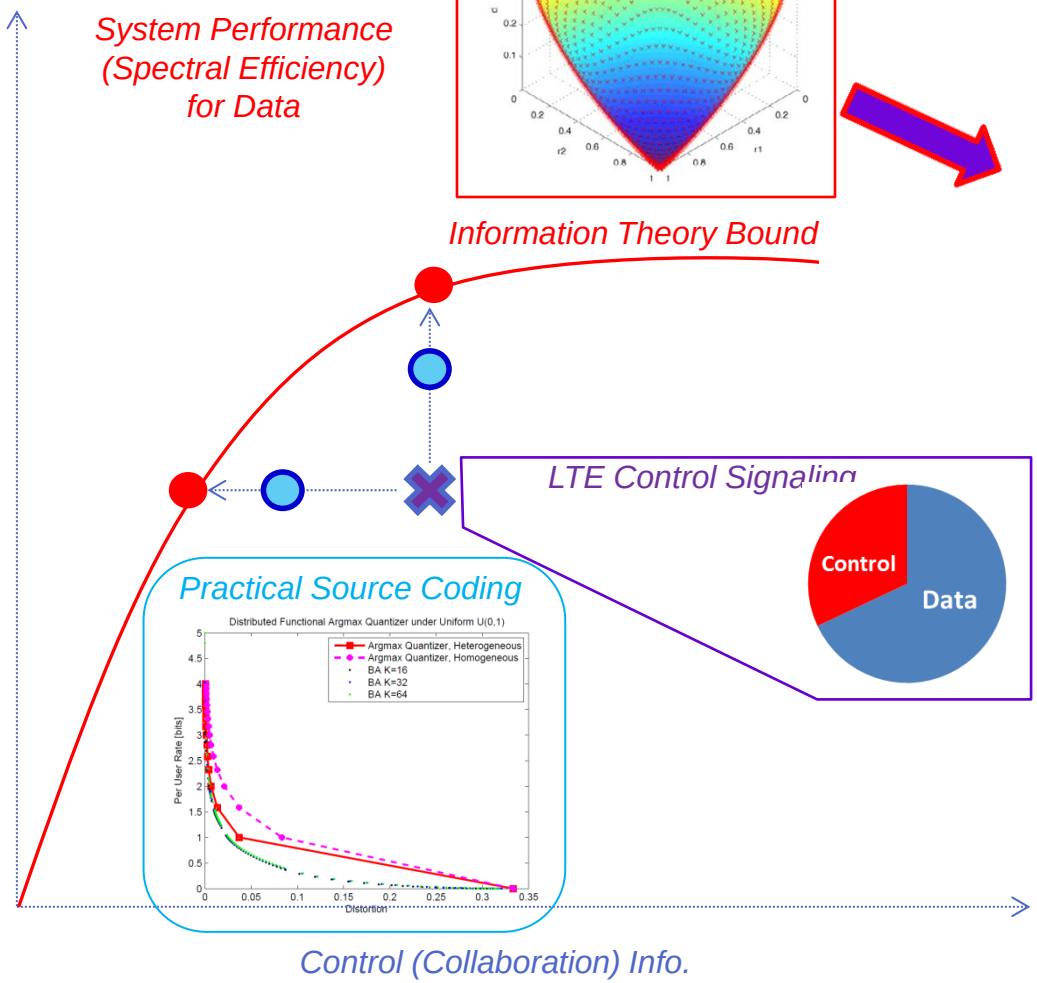
$$\min \mathbb{E}\left[d(T,\hat{T})\right]$$



Summary


Independent CEO
Rate Region
Calculation


Independent CEO with
Resource Allocation
Rate Distortion Curve based on 16 Leveled CQI
Rate Distortion Curve under Max(X1,X2)
Rate Distortion Curve under Argmax (X1,X2)
Rate Distortion Curve under max(X1,X2) & argmax(X1,X2)
Per User Rate [bits]
Normalized Distortion


System Performance
(Spectral Efficiency)
for Data
Information Theory Bound
LTE Control Signaling
Control
Data
Practical Source Coding
Distributed Functional Argmax Quantizer under Uniform U(0,1)
Argmax Quantizer, Heterogeneous
Argmax Quantizer, Homogeneous
BA K=16
BA K=32
BA K=64
Per User Rate [bits]
Distortion
Control (Collaboration) Info.

Thank you