Support Vector Machines and Speaker Verification

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Review of Speaker Verification

Introduction to Support Vector Machines

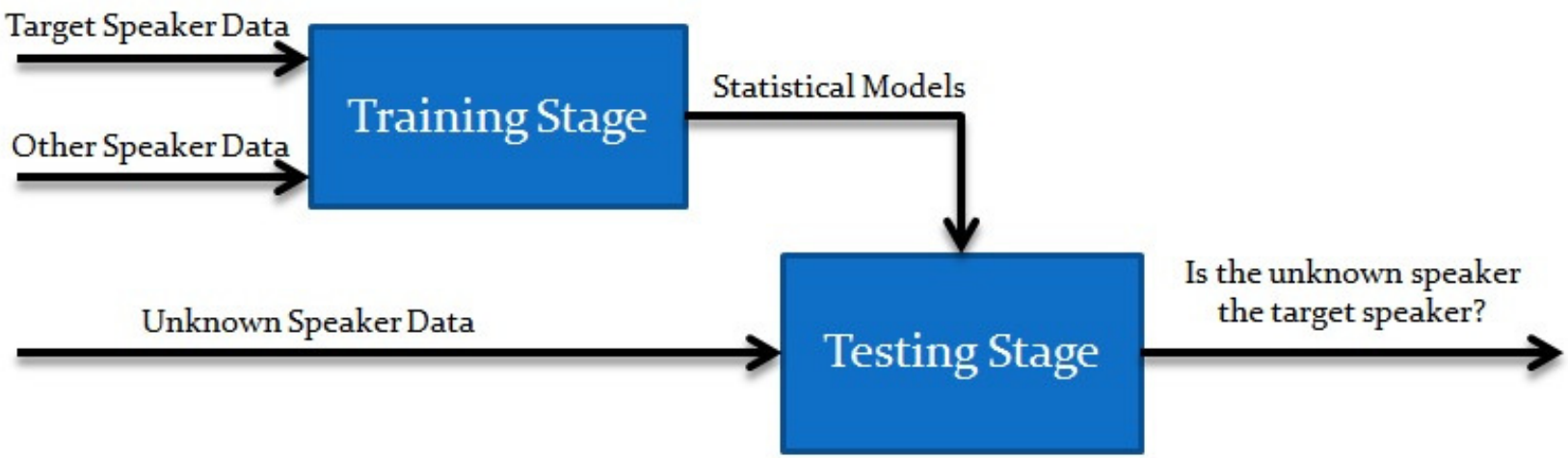
Derivation of SVM Equations

Soft Margin

Nonlinear Classification

SVMs in Speaker Verification
- Examples of Kernels Used in Speaker Verification
- How to Perform

Review of Speaker Verification



- Speaker Verification as discussed before was done using Gaussian Mixture Models
- One popular way to perform speaker verification is with Support Vector Machines

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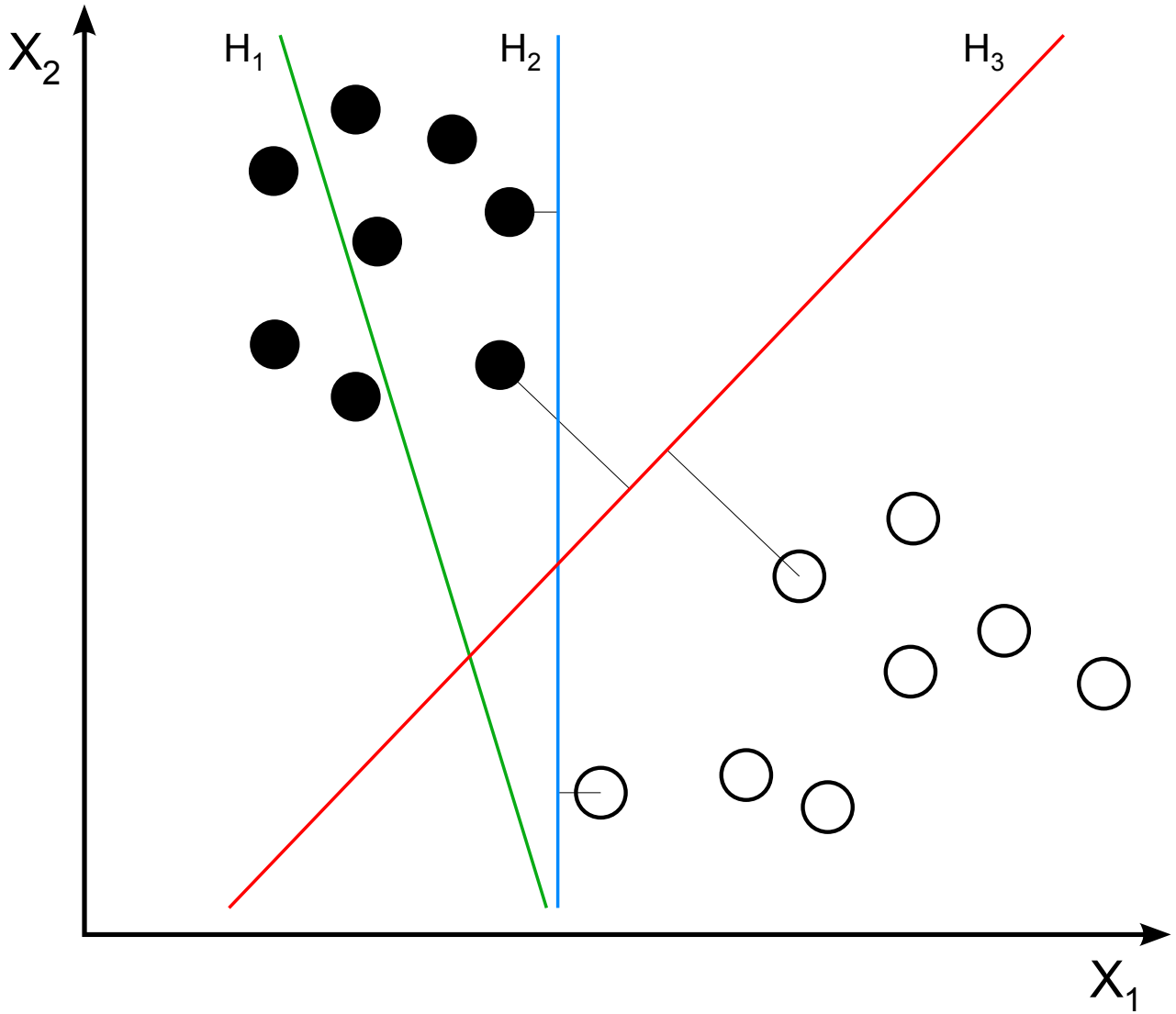
Motivation

- Training Data from two different classes are linearly separable.
- Want to classify unknown testing data into one of the classes
- Since training data is linearly separable, can create a line that separates the data
- Need to be able to classify testing data with minimum errors

A Pictoral Overview of Support Vector Machines



Kernel Functions

- Requires kernel trick to work with non-linearly separable training data
- Kernel Trick - Mapping items from a set S into an inner product space V without having to compute the mapping
- Data can be projected to higher dimensions using kernels
- In the linear case, kernel is:

$$k(x_i, x_j) = x_i \cdot x_j \tag{1}$$

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Formulation of the Problem

- Consider a set of data points of the form

$$\mathcal{S} = \{(\mathbf{x}_i, y_i) | \mathbf{x}_i \in \mathbb{R}^d, y_i \in \{-1, 1\}\} \tag{2}$$

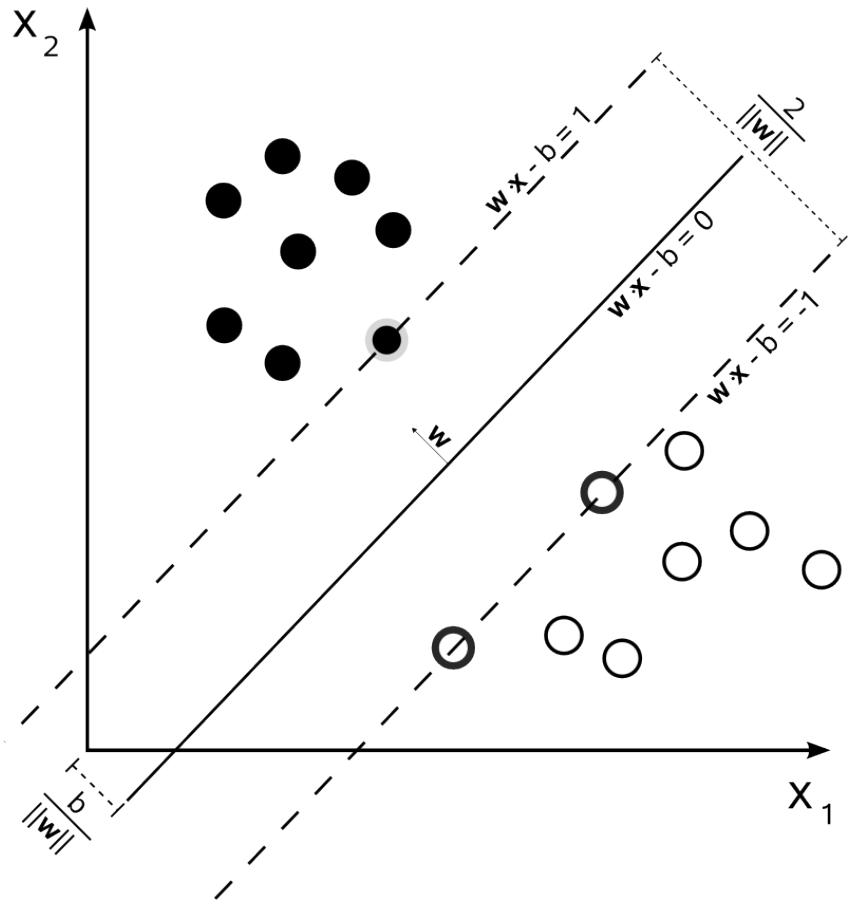
- y_i - indicator of which class a point $\mathbf{x}_i$ is of
- Need to a find a hyperplane that maximally separates data. Hyperplane of form:

$$\mathbf{w} \cdot \mathbf{x} - b = 0 \tag{3}$$

- Parameter $\frac{b}{||\mathbf{w}||}$ denotes affine offset from origin along normal vector $\mathbf{w}$

Selecting the Seperating Hyperplane

- First case: Linearly Separable Training Data
- Select two hyperplanes as shown
- Region between: "the margin"
- Hyperplanes are of form on picture
- Distance between the two is $\frac{2}{||\mathbf{w}||}$
- Thus, need to minimize $||\mathbf{w}||$



Formulating the Optimization Problem

- Want to prevent points from falling into the margin. Thus add constraints
- For $\mathbf{x}_i$ of the first class

$$\mathbf{w} \cdot \mathbf{x}_i - b \geq 1 \tag{4}$$

- For $\mathbf{x}_i$ of the second class

$$\mathbf{w} \cdot \mathbf{x}_i - b \leq -1 \tag{5}$$

- Or can be written together as

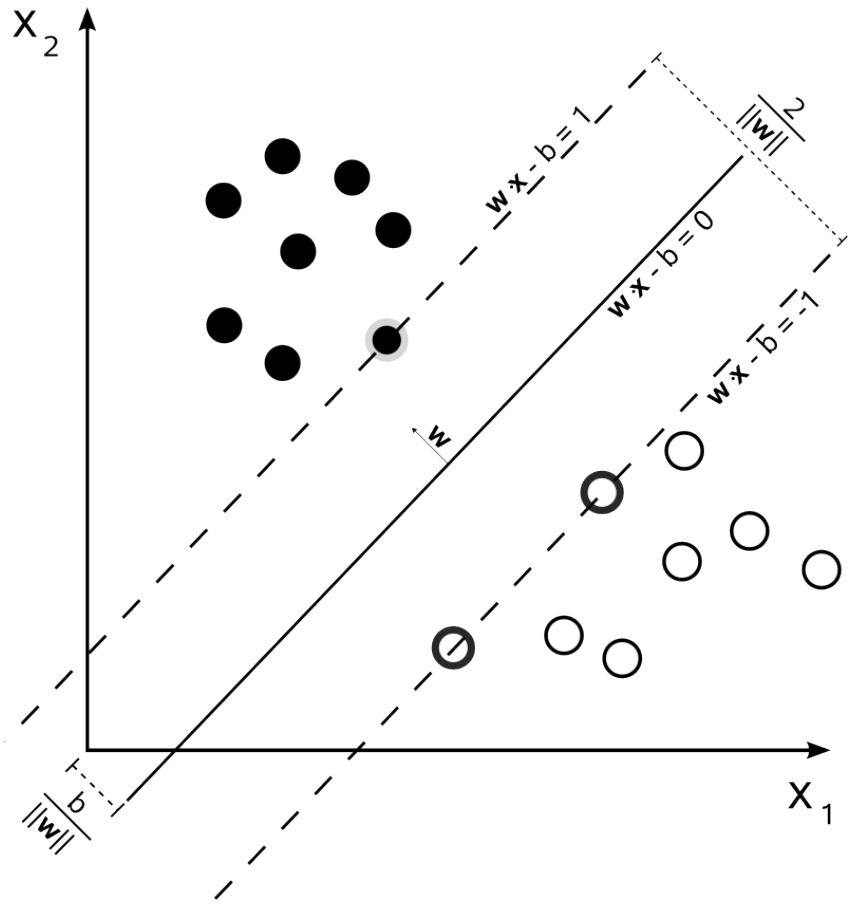
$$y_i(\mathbf{w} \cdot \mathbf{x}_i - b) \geq 1 \tag{6}$$

The Optimization Problem

$$\min_{\mathbf{w},b} \frac{1}{2}||\mathbf{w}||^2 \qquad (7)$$

subject to

$$y_i(\mathbf{w} \cdot \mathbf{x}_i - b) \geq 1 \qquad (8)$$

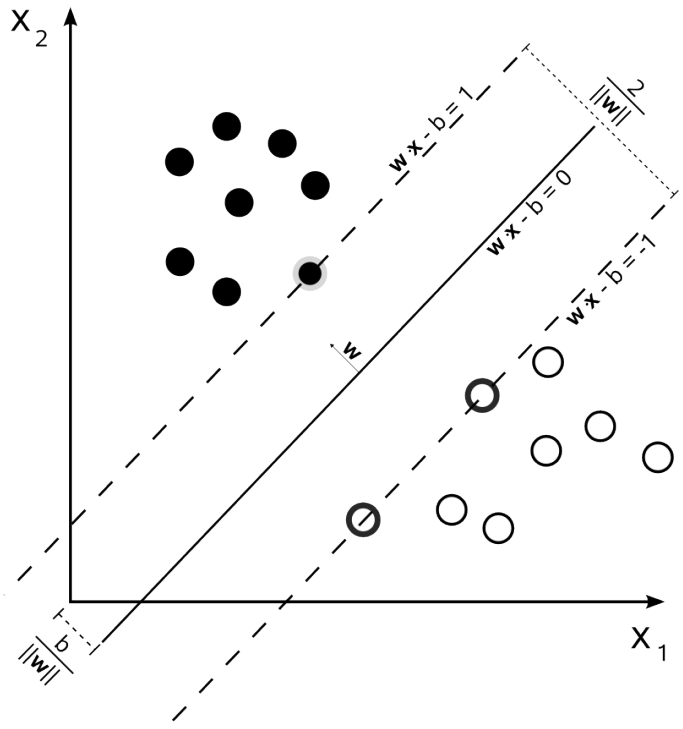


The Primal Problem

- The Lagrangian can be defined as

$$L_P := \frac{1}{2}||\mathbf{w}||^2 - \sum_{i=1}^{N} \alpha_i [y_i(\mathbf{w} \cdot \mathbf{x}_i - b) - 1] \tag{9}$$

- Primal Problem is convex since objective is convex and constraints are linear.



The Primal Problem

Solving the gradient gives us

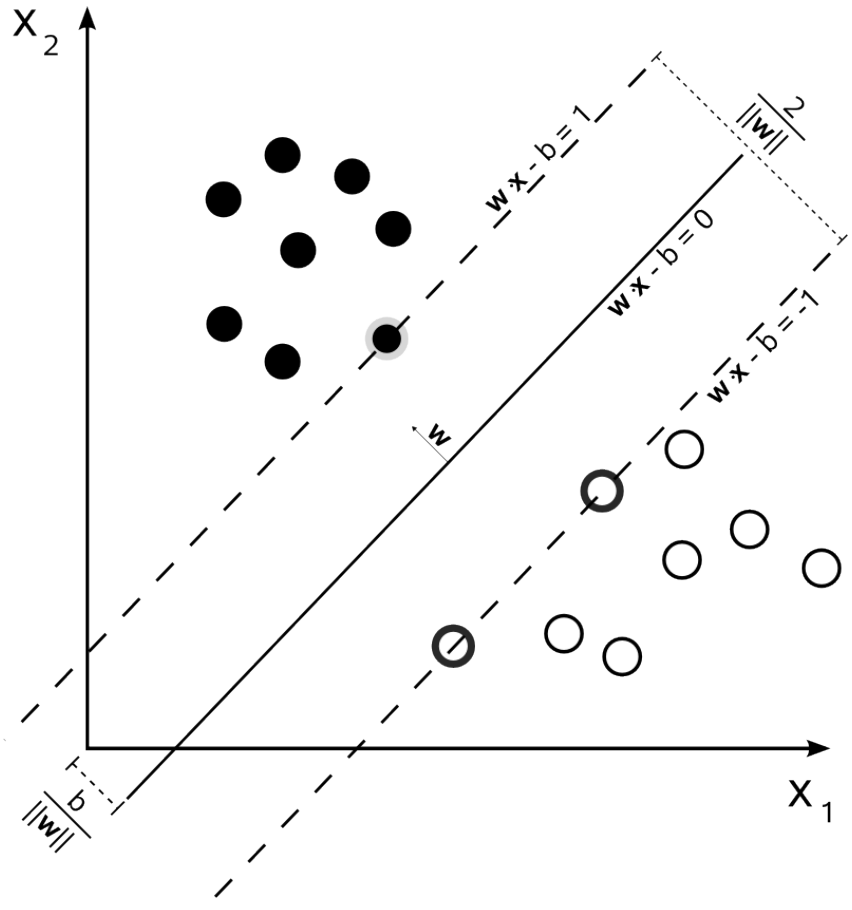
$$\mathbf{w} = \sum_i \alpha_i y_i \mathbf{x}_i \qquad (10)$$

$$\sum_i \alpha_i y_i = 0 \qquad (11)$$

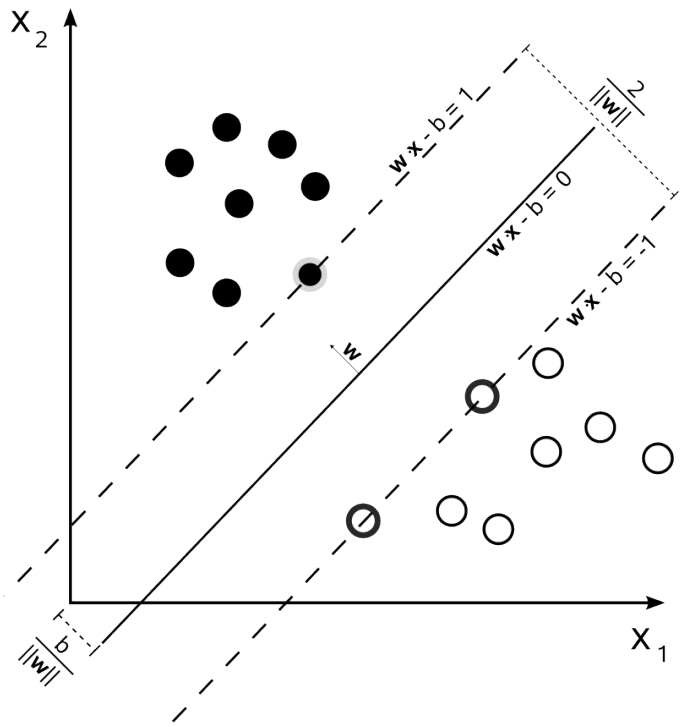
and

$$b = \frac{1}{N_{SV}} \sum_{i=1}^{N_{SV}} (\mathbf{w} \cdot \mathbf{x}_i - y_i) \qquad (12)$$

where i is taken over the support vectors (the vectors on the edge of the boundary)



The Dual Problem



- Since primal problem is convex, there is no duality gap
- Dual Lagrangian given as

$$L_D = \sum_i \alpha_i - \frac{1}{2} \sum_{i,j} \alpha_i \alpha_j y_i y_j \mathbf{x}_i \cdot \mathbf{x}_j \tag{13}$$

KKT Conditions

$$\frac{\partial}{\partial w_v} L_P = w_v - \sum_i \alpha_i y_i x_{iv} = 0 \quad v = 1, \ldots, d \tag{14}$$

$$\frac{\partial}{\partial b} L_P = -\sum_i \alpha_i y_i = 0 \tag{15}$$

$$y_i(\mathbf{w} \cdot \mathbf{x}_i + b) - 1 \geq 0 \quad i = 1, \ldots, l \tag{16}$$

$$\alpha_i \geq 0 \quad \forall i \tag{17}$$

$$\alpha_i(y_i(\mathbf{w} \cdot \mathbf{x}_i + b) - 1) = 0 \quad \forall i \tag{18}$$

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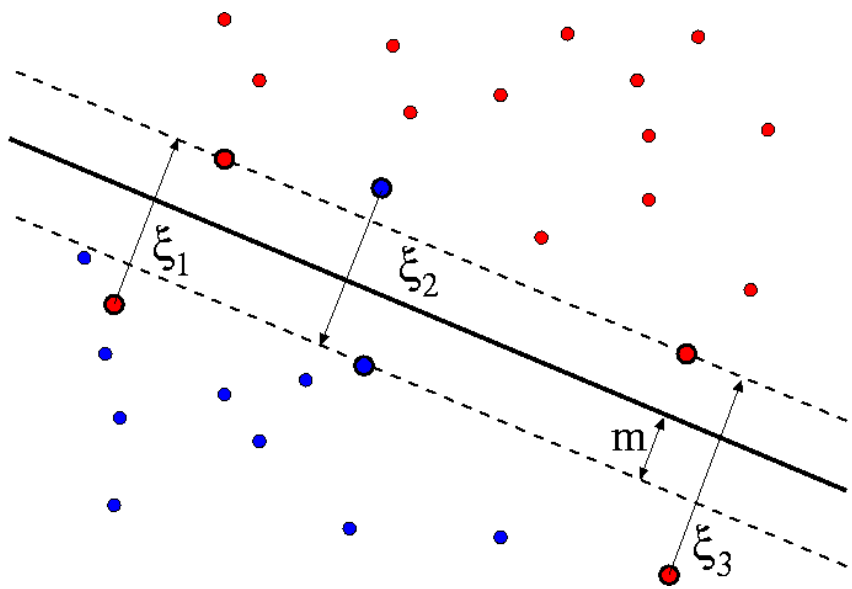
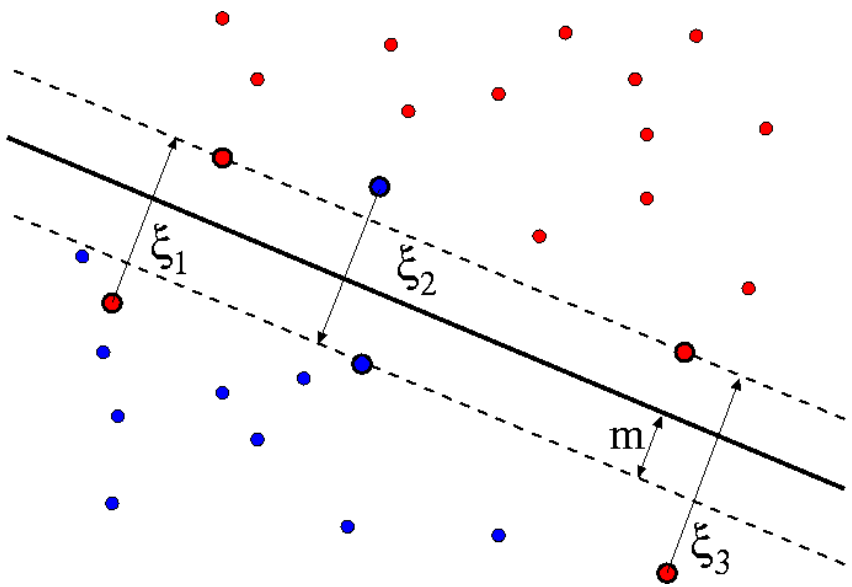
Unseparable Data

- Sometimes data from one class is mixed with data from other
- Can project with kernels but may overfit
- Want to allow for errors



Optimization Problem for Soft Margin



- Introduce linear penalty function featuring slack variable ξ_i which measure the degree of misclassification of the data
- Optimization problem becomes

$$\min_{\mathbf{w},\xi,b}\{\frac{1}{2}||\mathbf{w}||^2 + C\sum_{i=1}^{N}\xi_i\} \quad (19)$$

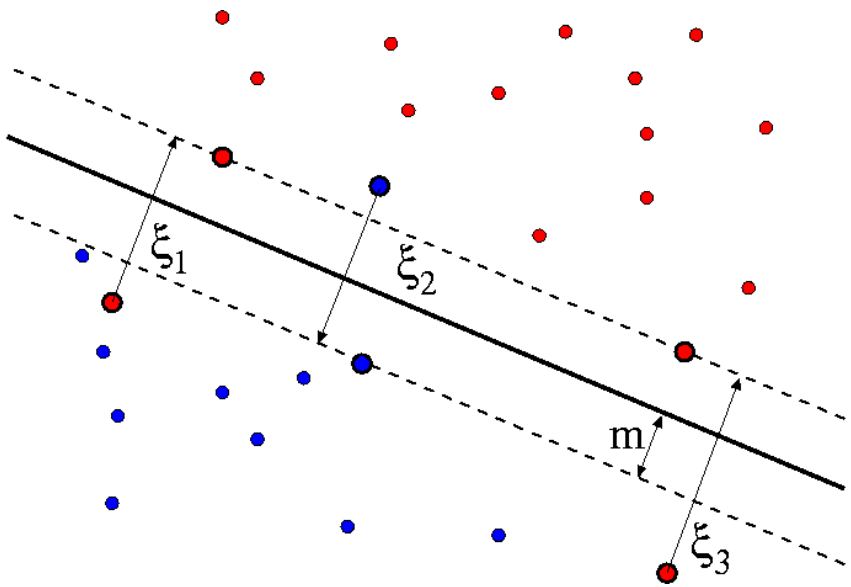
subject to

$$y_i(\mathbf{w}\cdot\mathbf{x}_i - b) \geq 1 - \xi_i \quad \xi_i \geq 0 \quad (20)$$

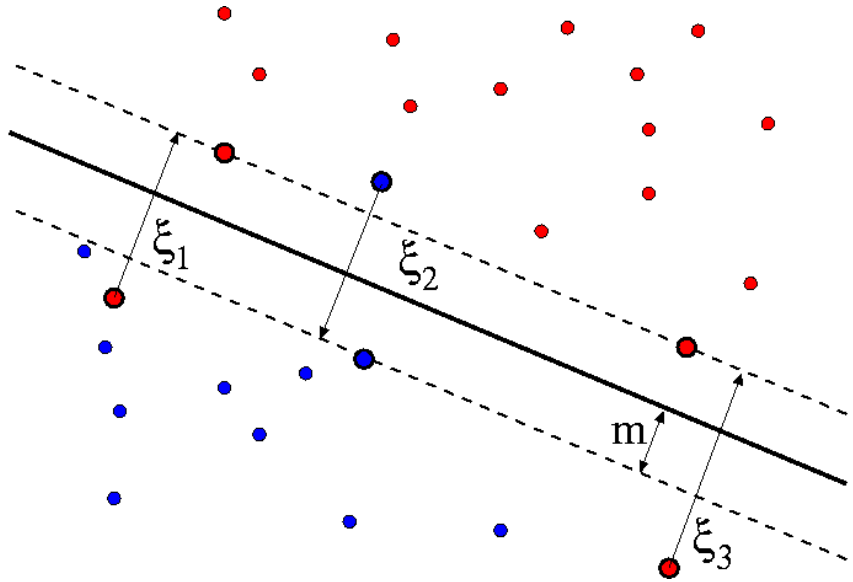
Primal Lagrangian

$$L_P = \frac{1}{2}||\mathbf{w}||^2 + C\sum_{i=1}^{n}\xi_i - \sum_{i=1}^{n}\alpha_i[y_i(\mathbf{w}\cdot\mathbf{x}_i - b) - 1 + \xi_i] - \sum_{i=1}^{n}\beta_i\xi_i \quad (21)$$

with $\alpha_i, \beta_i \geq 0$



Dual Lagrangian



$$L_D = \sum_{i=1}^{n} \alpha_i - \frac{1}{2} \sum_{i,j} \alpha_i \alpha_j y_i y_j \mathbf{x}_i \cdot \mathbf{x}_j \tag{22}$$

subject to

$$0 \leq \alpha_i \leq C \tag{23}$$

and

$$\sum_{i=1}^{n} \alpha_i y_i = 0 \tag{24}$$

KKT Conditions for Soft Margin

$$\frac{\partial L_P}{\partial w_v} = w_v - \sum_i \alpha_i y_i x_{iv} = 0 \tag{25}$$

$$\frac{\partial L_P}{\partial b} = -\sum_i \alpha_i y_i = 0 \tag{26}$$

$$\frac{\partial L_P}{\partial \xi_i} = C - \alpha_i - \mu_i = 0 \tag{27}$$

$$y_i(\mathbf{w} \cdot \mathbf{x}_i + b) - 1 + \xi_i \geq 0 \tag{28}$$

$$\xi_i \geq 0 \tag{29}$$

$$\alpha_i \geq 0 \tag{30}$$

$$\mu_i \geq 0 \tag{31}$$

$$\alpha_i[y_i(\mathbf{w} \cdot \mathbf{x}_i - b) - 1 + \xi_i] = 0 \tag{32}$$

$$\mu_i \xi_i = 0 \tag{33}$$

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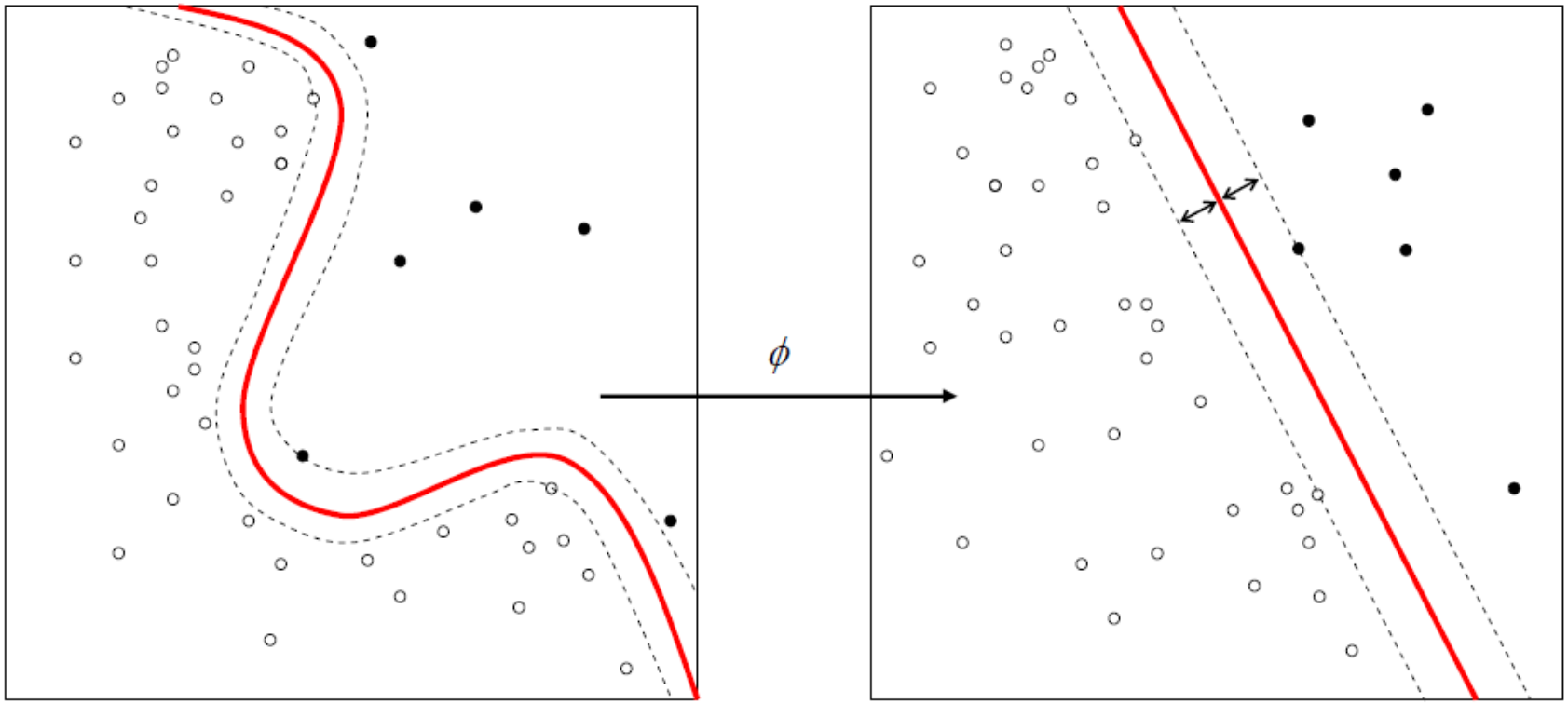
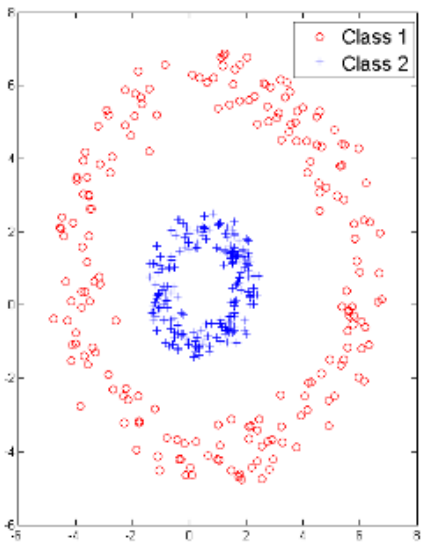
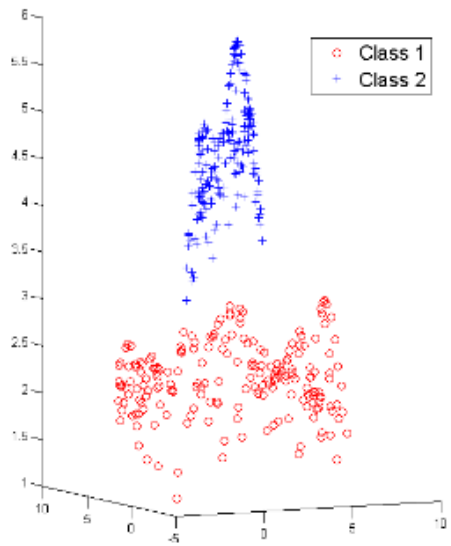


- Use Kernels to Transform Nonlinearly separable data into linearly separable data
- Project Data to Higher Dimensions

Kernel Function





- Kernel function is dot product of two vectors projected into another space.

$$K(\mathbf{x}_i, \mathbf{x}_j) = \phi((x)_i) \cdot \phi((x)_j) \tag{34}$$

- SVM equations only require dot product of two vectors. Can work with infinite dimensional projections

Calculating the Decision Boundary

- Can use same optimization as before, substituting $K(\mathbf{x}_i, \mathbf{x}_j)$ for $\mathbf{x}_i \cdot \mathbf{x}_j$
- New data points can be classified by following equation

$$f(x) = \sum_{i=1}^{N_S} \alpha_i y(i) K(\mathbf{s}_i, \mathbf{x}) + b \tag{35}$$

where $\mathbf{s}_i$ is the support vectors

Mercer's Condition

- Kernels must be positive semidefinite to obtain uniform convergence to a solution for all training sets
- For any $g(\mathbf{x})$ such that $|\int g(\mathbf{x})d\mathbf{x}| < \infty$

$$\int K(\mathbf{x}, \mathbf{y})g(\mathbf{x})g(\mathbf{y})d\mathbf{x}d\mathbf{y} \geq 0 \tag{36}$$

If Mercer Condition is Not Achieved

- Training data may cause Hessian to be indefinite
- No solution can be found generally
- May find solution for some set of training vectors

Examples of Kernels

- Polynomial

$$K(\mathbf{x}_i, \mathbf{x}_j) = (\mathbf{x}_i \cdot \mathbf{x}_j + 1)^d \tag{37}$$

- Gaussian Radial Basis Function

$$K(\mathbf{x}_i, \mathbf{x}_j) = e^{-\gamma ||\mathbf{x}_i - \mathbf{x}_j||^2} \tag{38}$$

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Typical Question

- What Kernel to Use?
- What Space to Model Data in?

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Radial Basis Functions

- Use standard radial basis function for kernel
- Run on MFCCs: each one a data point
- Subtract off means of MFCCs to normalize first

GMM Supervector Kernel

- MAP Adaptation of Means of UBM for each utterance is data point
- Compares parameters from unknown speaker to parameters from target and background speakers

$$K(utt_a, utt_b) = \sum_{i=1}^{N} w_i \mu_i^a \Sigma_i^{-1} \mu_i^b \tag{39}$$

- Current baseline SVM system for Speaker Verification (GMM-SVM based system)

Nuisance Attribute Projection

- Tries to project out subspaces that cause varability in the data.
- Nonlinear expansion of the GMM Supervector Kernel
- Compared to Factor Analysis, does not estimate variability

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How to Perform

- New SVM Decision boundary for each target speaker
- For MFCC-based kernels, target data are target MFCCs
- For GMM Supervector-based kernels, target data are adapted mean vectors from a UBM
- Background data are either background MFCCs or background adapted means

Sources for Pictures

- Slide 6: ZackWeinberg on Wikipedia adapting a picture by Cyc
- Slides 10, 12, 13, 14, 15: Cyc on Wikipedia
- Slides 18, 19, 20, 21: EMILeA-stat by Institut fr Statistik und Wirtschaftsmathematik (`http://emilea-stat.stochastik.rwth-aachen.de/cgi-bin/WebObjects/EMILeAstat.woa/wo/0.0.27.1.1.3.0`)
- Slide 24: Alisneaky on Wikipedia
- Slide 25: Tristan Fletcher, Support Vector Machines Explained, UCL. London, England